What’s wrong with uncertainty training courses?!

- Too abstract
- Too much mathematics
- Examples from the wrong field
- Examples don’t apply
- Too hard – or too easy
- Don’t introduce correlation
- I want to know how to solve my own (real) problem
What I’d like to do …

• Give everyone a one-to-one uncertainty training course
  • Set at their own level
  • With the amount of mathematics they are comfortable with (even if that’s more than I am comfortable with!)
  • With their own problem as the case study

• How close can we get to this ideal on a realistic training course?
  • Specialist training courses
  • Cover basics, but also some more advanced topics
  • Plenty of one-to-one time (long “breaks”, long “question and answer”)

• The main challenge for me …
  • My expertise is radiometry …
The first course (3rd/4th July)

Uncertainty Analysis for Earth Observation
Radiometric Instrument Calibration

• Runs: 3rd/4th July 2014 at NPL
• Now full up – but it’s worth signing the waiting list
• (Go to: http://www.npl.co.uk/events/3-4-jul-2014-uncertainties-for-earth-observation-training-course)
Developing the course

- Generic concepts
- Key case study: APEX instrument calibration (developed by Andreas Hueni)
Future plans

• Course on ‘products’
  • During 2015
  • Will need a case study for this too!
  • (And maybe several)

• NPL E-learning Course
  • (see: http://www.npl.co.uk/commercial-services/products-and-services/training/e-learning/introduction-to-uncertainty/)
Understanding the Law of Propagation of Uncertainties
The GUM

The guide to the expression of uncertainty in measurement (GUM)

- The foremost authority and guide to the expression and calculation of uncertainty in measurement science
- Written by the JCGM and BIPM
- Covers a wide number of applications
- Technical with formal mathematics

Uncertainty – Error - Correction

**Uncertainty**

Describes the spread
drawn from a probability distribution described by uncertainty

**Error**

Difference to the (unknowable) true value

Residual, uncorrectable, unknown error

**Correction**

Known offset from true value

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Sources of uncertainty

- Error is caused by several effects
  - Some known (correctable offsets) which can and should be corrected
  - Many unknown effects
    - Each is a “source of uncertainty”
    - Things like alignment, noise, temperature sensitivity, calibration, etc.
    - Each has its own probability distribution described by an associated uncertainty
    - When a measurement is made the unknown error comes from all these effects, some one way, some the other – each a draw from that probability distribution
  - Final error ( unknowable) is a combination of all these
  - The final error is a draw from the combined probability distribution described by the combined uncertainty

Not GUM

Uncertainty analysis

GUM
### Systematic and random effects:
Lamp used 5 times to calibrate irradiance instrument

<table>
<thead>
<tr>
<th>EFFECT</th>
<th>SYSTEMATIC</th>
</tr>
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<tbody>
<tr>
<td>Calibration of reference</td>
<td>Yes</td>
</tr>
<tr>
<td>Alignment</td>
<td>If not realigned</td>
</tr>
<tr>
<td>Noise</td>
<td>No</td>
</tr>
<tr>
<td>Lamp current setting</td>
<td>Probably – if constant</td>
</tr>
<tr>
<td>Lamp current stability</td>
<td>Probably not</td>
</tr>
<tr>
<td>Temperature sensitivity</td>
<td>Depends on how much temperature is changing</td>
</tr>
</tbody>
</table>

Effects are random or systematic depending on the measurement process itself.
## Systematic and random effects: Lamp measured 5 times (continued)

<table>
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\[
S, u(S) \quad R_i, u(R_i)
\]

\[
E_i = E_{\text{True}} + S + R_i
\]

\[
u^2(E_i) = u^2(S) + u^2(R_i)
\]
Uncertainty types

There are two methods for estimating uncertainties:

**Type-A:**
uncertainty estimates using statistics i.e. by taking multiple readings and using that information

**Type-B:**
uncertainty estimates from any other information, e.g. past experience, calibration certificates, etc.
8 steps to an uncertainty budget

- Understanding the problem
  - Step 1: Describing the Traceability Chain
  - Step 2: Writing down the calculation equations
  - Step 3: Considering the sources of uncertainty
- Determining the formal relationships
  - Step 4: Creating the measurement equation
  - Step 5: Determining the sensitivity coefficients
  - Step 6: Assigning uncertainties
- Propagating the uncertainties
  - Step 7: Combining and propagating uncertainties
  - Step 8: Expanding uncertainties
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An unbroken chain
Step 1: Describing the Traceability Chain

SI

PTB calibration of source

Calibration of spectrometer against source

Spectral radiance of sphere calibrated with spectrometer

Sphere radiance

NPL calibration of filters

Filter Spectral Transmittance

Computed radiance for the sphere – filter combination

instrument radiometric gain

Radiance of observed scene

Calibration Home Base at DLR
Step 2: Writing down the calculation equations

\[ G_{SVC} = \frac{L_{\text{RASTA}}}{DN_{\text{RASTA}}} \]

\[ L_{\text{sphere}} = DN_{\text{sphere}} G_{\text{SVC}} \]

FIGURE 1. Mechanical set-up of RASTA (left): (1) Reflectance panel, (2) lamp housing, (3) translation stage, (4) mounting adapter, (5) radiometer holder, (6) radiometer, (7) sensor fitting. Yellow: beam illuminating reflectance panel, blue: field of view of radiometers. RASTA after its alignment for calibration in the 0°-45° viewing geometry at the SRCF of PTB (right).
Step 2: Writing down the calculation equations

\[ L_{\text{sphere}} = D N_{\text{sphere}} G_{\text{SVC}} \]

\[ L_{\text{sph-filt}} = L_{\text{sphere}} \tau_{\text{filter}} \]

\[ L_{\text{scene}} = G_{\text{APEX}} D N_{\text{APEX,scene}} \]

\[ D N_{\text{APEX,scene}} = D N_{\text{APEX,scene,light}} - D N_{\text{APEX,scene,dark}} \]

\[ G_{\text{APEX}} = L_{\text{sph-filt}} / D N_{\text{APEX,cal}} \]

\[ D N_{\text{APEX,cal}} = D N_{\text{APEX,cal,light}} - D N_{\text{APEX,cal,dark}} \]
Step 3: Considering the sources of uncertainty

1. From the calculation equation

2. From assumptions in the comparison made
Step 3: Considering the sources of uncertainty

\[ G_{\text{SVC}} = \frac{L_{\text{RASTA}}}{DN_{\text{RASTA}}} \]

\[ L_{\text{sphere}} = DN_{\text{sphere}} G_{\text{SVC}} \]

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Step 3: Considering the sources of uncertainty

\[ L_{\text{sphere}} = D N_{\text{sphere}} G_{\text{SVC}} \]

- Uncertainties associated with the terms in the calculation equation:
  - Signal: Noise in light signal, noise in dark signal
  - SVC gain: Calculated from the previous step (its calibration against RASTA)

- Uncertainties associated with the assumptions in the comparison:
  - Change in SVC between calibration and use
  - External stray light influencing the SVC differently
  - Internal stray light influencing the SVC differently
  - Environmental sensitivities of the SVC (temperature, pressure, humidity)
  - Linearity of the SVC
8 steps to an uncertainty budget

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Step 4: Creating the measurement equation

\[ L_{sphere} = D N_{sphere} G_{SVC} \]

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  - Signal: Noise in light signal, noise in dark signal
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  - Internal stray light influencing the SVC differently
  - Environmental sensitivities of the SVC (temperature, pressure, humidity)
  - Linearity of the SVC

\[ L_{sphere} = \left( D N_{sphere \_l} - D N_{sphere \_d} \right) G_{SVC} K_{SVC\_dft} K_{stray} K_{stray\_in} K_{temp} K_{lin} \]
Step 5: Determining the sensitivity coefficients

\[ L_{\text{sphere}} = (DN_{\text{sphere\_l}} - DN_{\text{sphere\_d}})G_{\text{SVC}}K_{\text{SVC\_dft}}K_{\text{stray}}K_{\text{stray\_in}}K_{\text{temp}}K_{\text{lin}} \]

\[ (DN)_{s} = \frac{1}{N} \sum_{i=1}^{N} (DN)_{\text{light},i} - \frac{1}{M} \sum_{i=1}^{M} (DN)_{\text{dark},j} \]

\[ c_{(DN)_{\text{light},i}} = \frac{\partial (DN)}{\partial (DN)_{\text{light},i}} = \frac{1}{N}, \quad i = 1, K, N \]

\[ c_{(DN)_{\text{dark},j}} = \frac{\partial (DN)}{\partial (DN)_{\text{dark},j}} = \frac{1}{M}, \quad j = 1, K, M \]

\[ u_{(DN)}^2 = \sum_{i=1}^{N} \left( \frac{1}{N} \right)^2 u_{(DN)_{\text{light},i}}^2 + \sum_{j=1}^{M} \left( \frac{1}{M} \right)^2 u_{(DN)_{\text{dark},j}}^2 \]

\[ u_{(DN)}^2 = \left( \frac{u_{(DN)_{\text{light}}}}{\sqrt{N}} \right)^2 + \left( \frac{u_{(DN)_{\text{dark}}}}{\sqrt{M}} \right)^2 \]
Step 5: Determining the sensitivity coefficients

\[ L_{\text{sphere}} = \left( D N_{\text{sphere}} \right) G_{\text{SVC}} K_{\text{SVC\_dft}} K_{\text{stray}} K_{\text{stray\_in}} K_{\text{temp}} K_{\text{lin}} \]

\[ C_{(DN)} = \frac{\partial L_{\text{sphere}}}{\partial (DN)} = \frac{L_{\text{sphere}}}{(DN)} \]

\[ C_{G_{\text{SVC}}} = \frac{\partial L_{\text{sphere}}}{\partial G_{\text{SVC}}} = \frac{L_{\text{sphere}}}{G_{\text{SVC}}} \]

\[ C_{K_{i}} = \frac{\partial L_{\text{sphere}}}{\partial K_{i}} = \frac{L_{\text{sphere}}}{K_{i}} \]

\[ \left( \frac{u_{L_{\text{sph}}}}{L_{\text{sph}}} \right)^2 = \left( \frac{u_{(DN)}}{(DN)} \right)^2 + \left( \frac{u_{G_{\text{svc}}}}{G_{\text{svc}}} \right)^2 + \left( \frac{u_{K_{\text{svc\_dft}}}}{K_{\text{svc\_dft}}} \right)^2 + K \]
Step 6: Assigning uncertainties

\[ L_{\text{sphere}} = \left( D N_{\text{sphere}} \right) G_{\text{SVC}} K_{\text{SVC\_dft}} K_{\text{stray}} K_{\text{stray\_in}} K_{\text{temp}} K_{\text{lin}} \]
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The Law of Propagation of Uncertainties

\[ u_c^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \]
Things you might already “know”

\[
u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)
\]

Has a sensitivity coefficient
Adding in quadrature (% or units)

Averages reduce by \(\frac{1}{\sqrt{n}}\)

This term is to do with correlation
The Law of Propagation of Uncertainties

\[ u_c^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \]
Correlation: Type A and Type B methods

Correlation occurs when there is something in common between the different quantities, or between the measured values being combined.

Type A: From the data
Type B: From knowledge

\[ E_i = E_{\text{True}} + S + R_i \]

This is where the correlation comes from!

Systematic Effects!
Correlation: Type A and Type B methods

Correlation occurs when there is something in common between the different quantities, or between the measured values being combined.

Type A: From the data

\[
 r(x, y) = \frac{1}{n-1} \sum_{i=1}^{n} \left( \frac{X_i - \bar{X}}{s_X} \right) \left( \frac{Y_i - \bar{Y}}{s_Y} \right)
\]

\[
 u(x, y) = u(x)u(y)r(x, y)
\]

\[
 u_c^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)
\]
Correlation: Type A and Type B methods

Correlation occurs when there is something in common between the different parameters, or between the measurements being combined.

Type B: From knowledge

\[ E_i = E_{\text{True}} + S + R_i \]

This is where the correlation comes from!

Systematic Effects!
Averaging partially correlated data

\[ E_i = E_{\text{True}} + S + R_i \]

\[ \frac{\partial E_M}{\partial E_1} = \frac{\partial E_M}{\partial E_2} = \frac{\partial E_M}{\partial E_3} = \frac{1}{3} \]

\[ E_M = \frac{E_1 + E_2 + E_3}{3} \]

\[ u(E_i) = u^2(S) + u^2(R_i) \]

\[ u(E_i, E_j) = u^2(S); \quad i \neq j \]

\[ u_c^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \]

\[ u^2(E_M) = 3 \left( \frac{1}{3} \right)^2 u^2(S) + 3 \left( \frac{1}{3} \right)^2 u^2(R_i) + 3 \times 2 \times \left( \frac{1}{3} \right)^2 u^2(S) \]

\[ u^2(E_M) = (3 + 6) \left( \frac{1}{3} \right)^2 u^2(S) + 3 \left( \frac{1}{3} \right)^2 u^2(R_i) \]

\[ u^2(E_M) = u^2(S) + \left( \frac{u(R_i)}{\sqrt{3}} \right)^2 \]
Averaging partially correlated data

\[ E_i = E_{\text{True}} + S + R_i \]

\[ E_M = \frac{E_1 + E_2 + E_3}{3} \]

\[ \frac{\partial E_M}{\partial E_1} = \frac{\partial E_M}{\partial E_2} = \frac{\partial E_M}{\partial E_3} = \frac{1}{3} \]

\[ E_M = \frac{3E_{\text{True}}}{3} + \frac{3S}{3} + \frac{R_1 + R_2 + R_3}{3} \]

\[ u^2(E_M) = u^2(S) + \left( \frac{u(R_i)}{\sqrt{3}} \right)^2 \]
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\[
S, u(S), R_i, u(R_i)
\]

\[
E_i = E_{\text{True}} + S + R_i
\]

\[
u^2(E_i) = u^2(S) + u^2(R_i)
\]

\[
u^2(E_M) = u^2(S) + \left(\frac{u(R_i)}{\sqrt{n}}\right)^2
\]
Yes, Emma ... But ...
EO is different!

- You can’t write simple equations like that!
- We lose our traceability on launch!
- We can’t define our measurement model!
I know ...

Launch and post-launch ageing effects

- Preflight characterisation of instrument angles
- Preflight preflight BRDF determination
- Preflight calibration of diffuser monitoring system

Instrument angles

On-board diffuser monitoring

Diffuser BRDF and radiance factor

Solar distance and angles

Diffuser radiance

Measured instrument signal

Instrument radiance gain coefficients
The first course (3<sup>rd</sup>/4<sup>th</sup> July)

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