National Physical Laboratory | Centre for Carbon Measurement

Uncertainty Training

Emma Woolliams

4th June 2014

Metrology for Earth Observation and Climate
http://www.emceoc.org

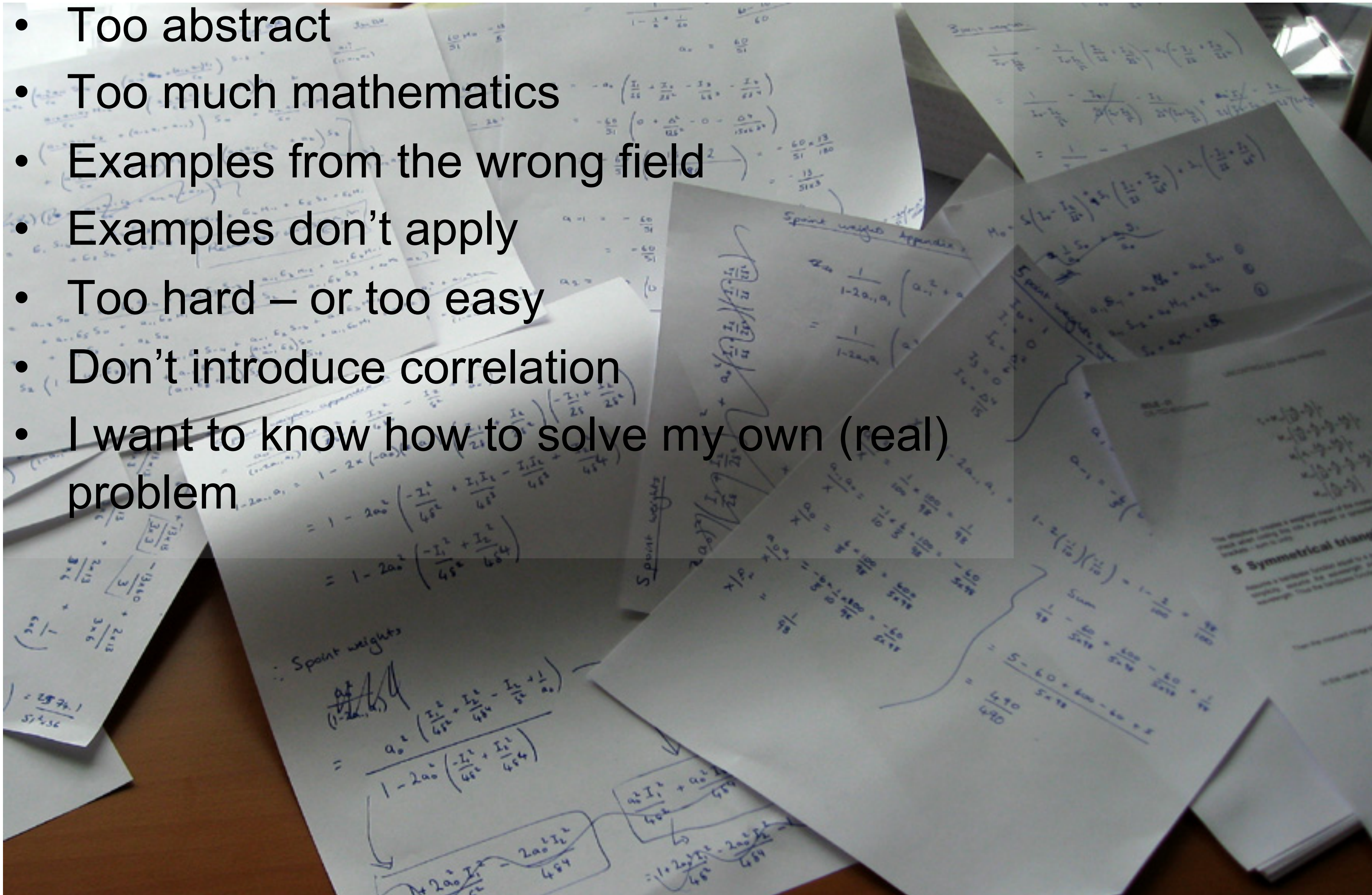
EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

EURAMET
European Association of National Metrology Institutes

What’s wrong with uncertainty training courses?!

- Too abstract
- Too much mathematics
- Examples from the wrong field
- Examples don’t apply
- Too hard – or too easy
- Don’t introduce correlation
- I want to know how to solve my own (real) problem



What I'd like to do …

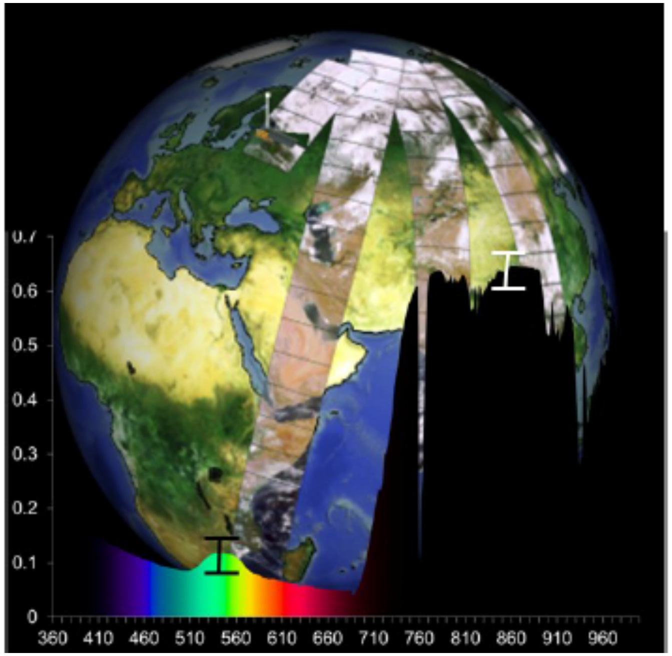
- Give everyone a one-to-one uncertainty training course
  - Set at their own level
  - With the amount of mathematics they are comfortable with (even if that's more than I am comfortable with!)
  - With their own problem as the case study

- How close can we get to this ideal on a realistic training course?
  - Specialist training courses
  - Cover basics, but also some more advanced topics
  - Plenty of one-to-one time (long "breaks", long "question and answer")

- The main challenge for me …
  - My expertise is radiometry …

The first course (3rd/4th July)

Uncertainty Analysis
for **Earth Observation**
Radiometric Instrument Calibration



- Runs: 3rd/4th July 2014 at NPL
- Now full up – but it's worth signing the waiting list
- (Go to: http://www.npl.co.uk/events/3-4-jul-2014-uncertainties-for-earth-observation-training-course)

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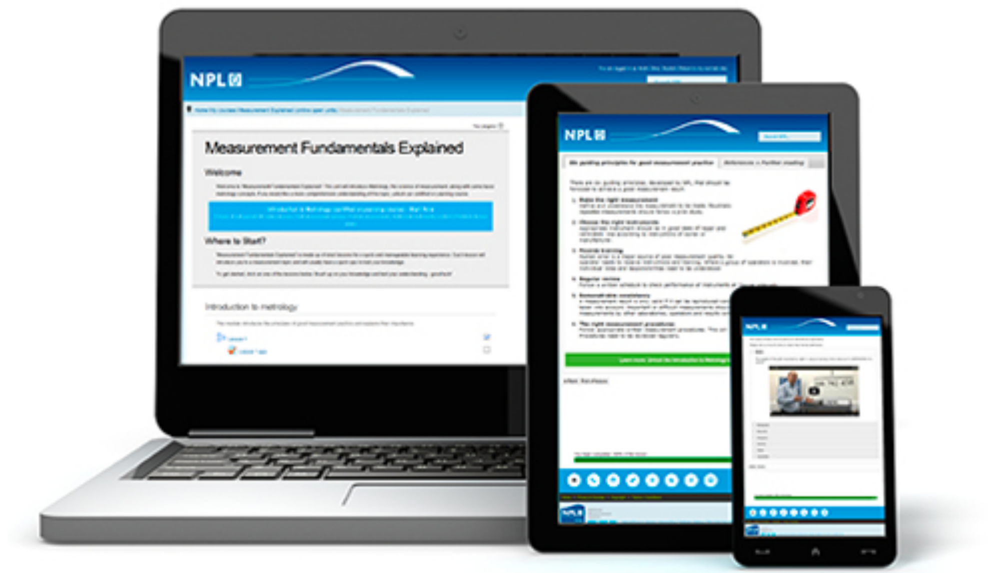
Developing the course

- Generic concepts
- Key case study: APEX instrument calibration (developed by Andreas Hueni)

Future plans

- Course on ‘products’
  - During 2015
  - Will need a case study for this too!
  - (And maybe several)

- NPL E-learning Course
  - (see: http://www.npl.co.uk/commercial-services/products-and-services/training/e-learning/introduction-to-uncertainty/)



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Understanding the Law of Propagation of Uncertainties

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The GUM


BIPM

JCGM 100:2008
GUM 1995 with minor corrections

Evaluation of measurement data — Guide to the expression of uncertainty in measurement

Évaluation des données de mesure — Guide pour l'expression de l'incertitude de mesure

First edition September 2008

ΜΕΤΡΩ ΧΡΩ

The guide to the expression of uncertainty in measurement (GUM)

- The foremost authority and guide to the expression and calculation of uncertainty in measurement science
- Written by the JCGM and BIPM
- Covers a wide number of applications
- Technical with formal mathematics

http://www.bipm.org/en/publications/guides/gum.html

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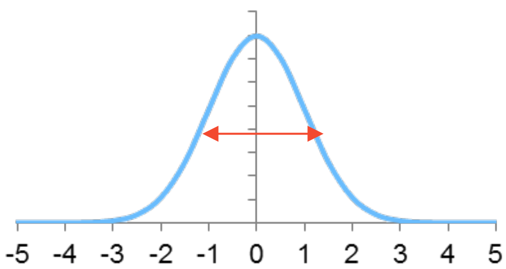
Uncertainty – Error - Correction

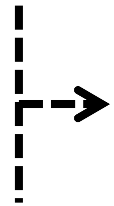
Uncertainty



Describes the spread

Drawn from a probability distribution described by uncertainty

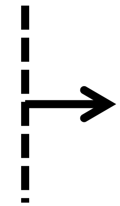
Error



Difference to the (unknowable) true value

Residual, uncorrectable, unknown error

Correction



Known offset from true value

Sources of uncertainty

- Error is caused by several effects
  - Some known (correctable offsets) which can and should be corrected
  - Many unknown effects
    - Each is a “source of uncertainty”
    - Things like alignment, noise, temperature sensitivity, calibration, etc.
    - Each has its own probability distribution described by an associated uncertainty
    - When a measurement is made the unknown error comes from all these effects, some one way, some the other – each a draw from that probability distribution
    - Final error (unknowable) is a combination of all these
    - The final error is a draw from the combined probability distribution described by the combined uncertainty

Systematic and random effects:

Lamp used 5 times to calibrate irradiance instrument

EFFECT	SYSTEMATIC
Calibration of reference	Yes
Alignment	If not realigned
Noise	No
Lamp current setting	Probably – if constant
Lamp current stability	Probably not
Temperature sensitivity	Depends on how much temperature is changing

Effects are random or systematic depending on the measurement process itself

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Systematic and random effects: Lamp measured 5 times (continued)

Systematic effects	Random effects
Reference calibration	Noise
Alignment	Lamp current fluctuation
Lamp current setting	
Temperature sensitivities	

$$S, u(S)$$

$$R_i, u(R_i)$$

$$E_i = E_{\text{True}} + S + R_i$$

$$u^2(E_i) = u^2(S) + u^2(R_i)$$

Uncertainty types

There are two methods for estimating uncertainties:

Type-A:

uncertainty estimates using statistics i.e. by taking multiple readings and using that information

Type-B:

uncertainty estimates from any other information, e.g. past experience, calibration certificates, etc.

8 steps to an uncertainty budget

- Understanding the problem
  - Step 1: Describing the Traceability Chain
  - Step 2: Writing down the calculation equations
  - Step 3: Considering the sources of uncertainty
- Determining the formal relationships
  - Step 4: Creating the measurement equation
  - Step 5: Determining the sensitivity coefficients
  - Step 6: Assigning uncertainties
- Propagating the uncertainties
  - Step 7: Combining and propagating uncertainties
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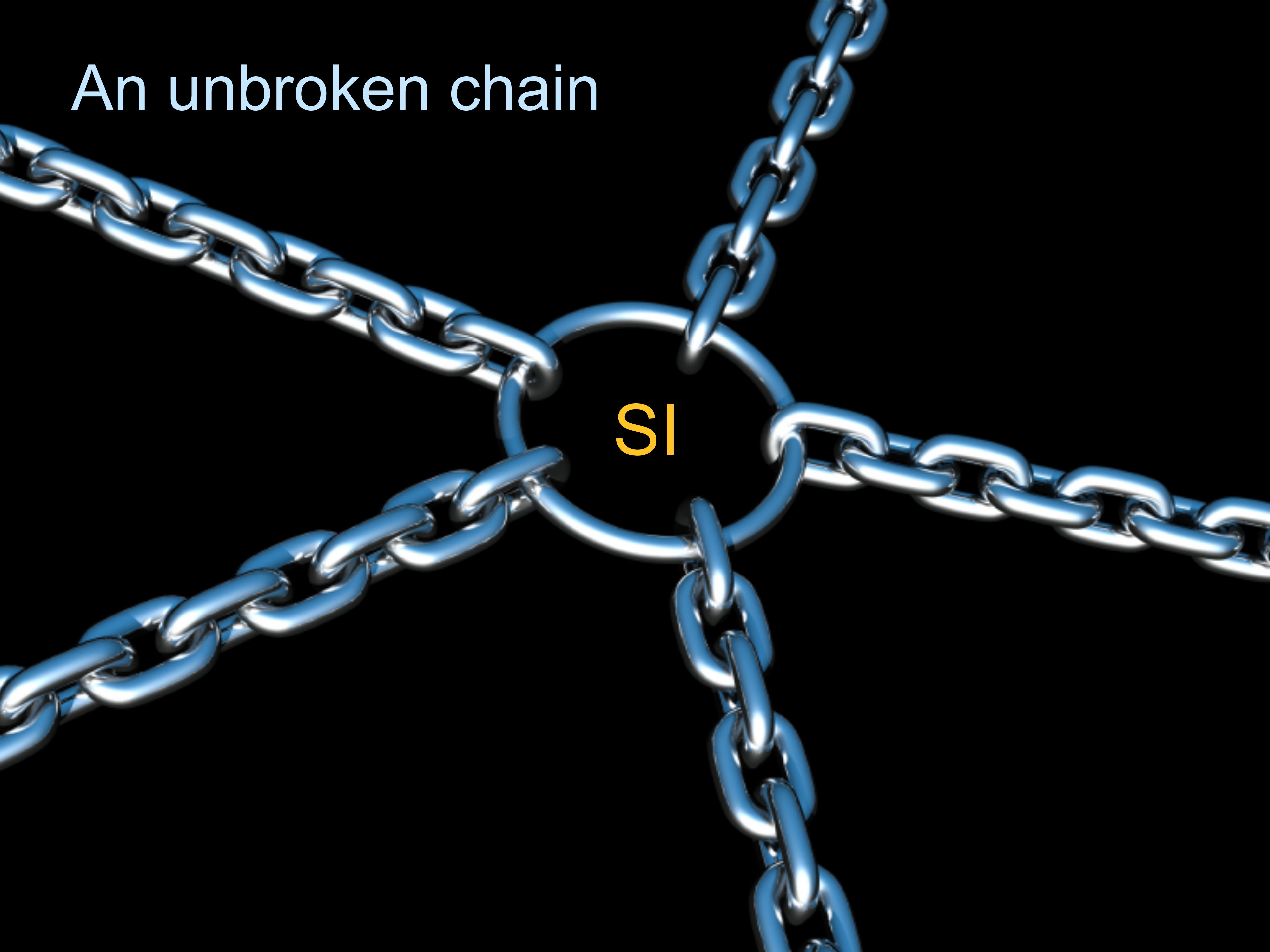
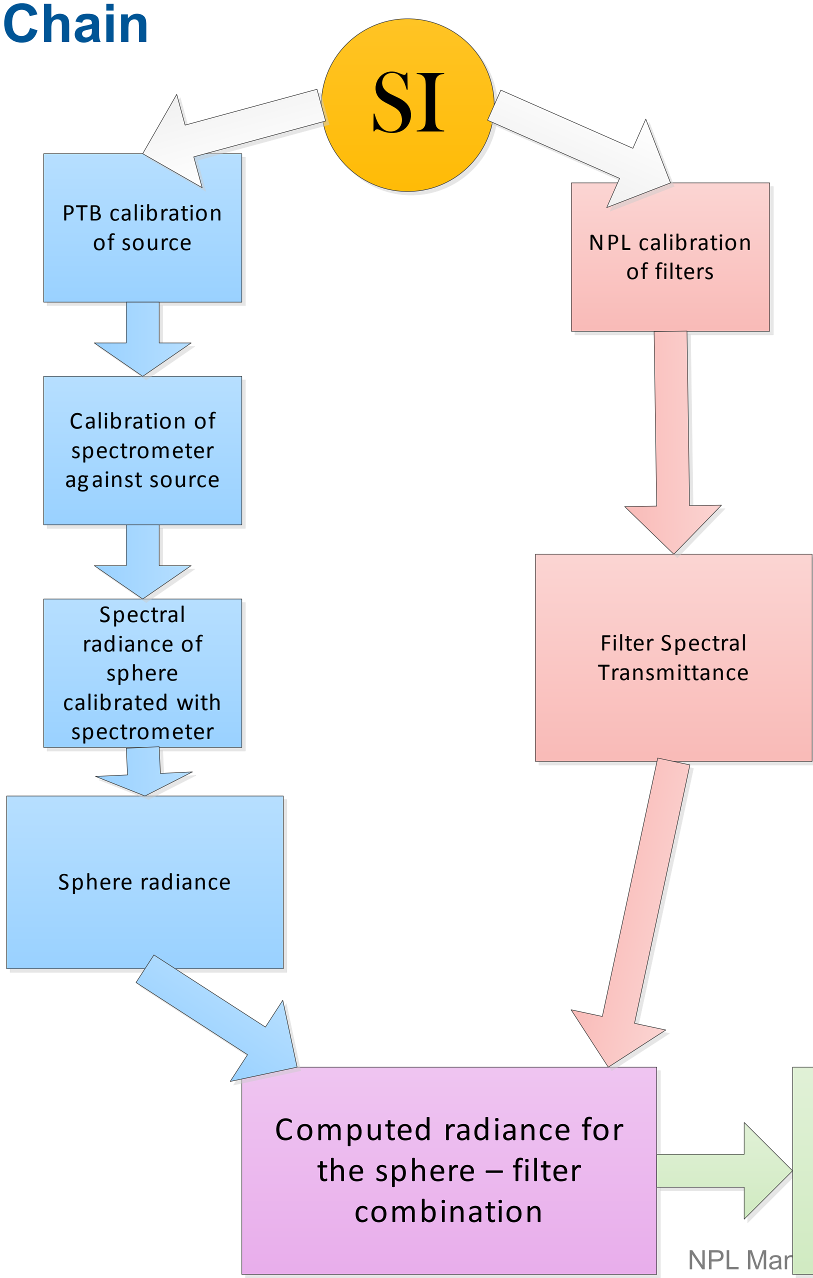
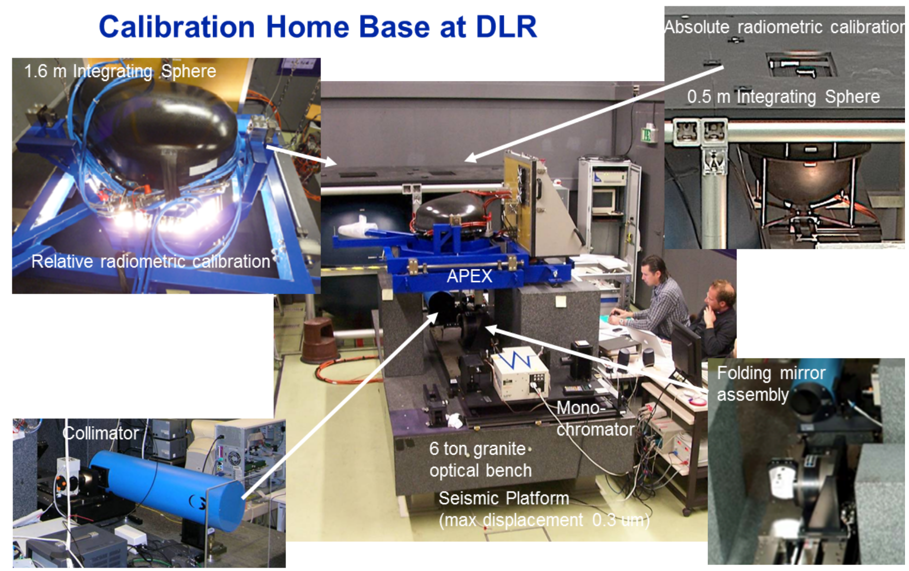
An unbroken chain



Step 1: Describing the Traceability Chain





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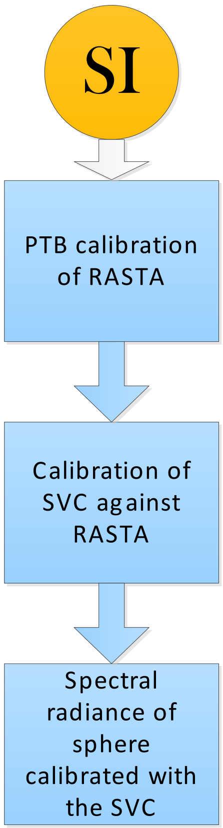
Step 2: Writing down the calculation equations



$$G_{\mathrm{SVC}} = \frac{L_{\mathrm{RASTA}}}{DN_{\mathrm{RASTA}}}$$

$$L_{\mathrm{sphere}} = DN_{\mathrm{sphere}} G_{\mathrm{SVC}}$$

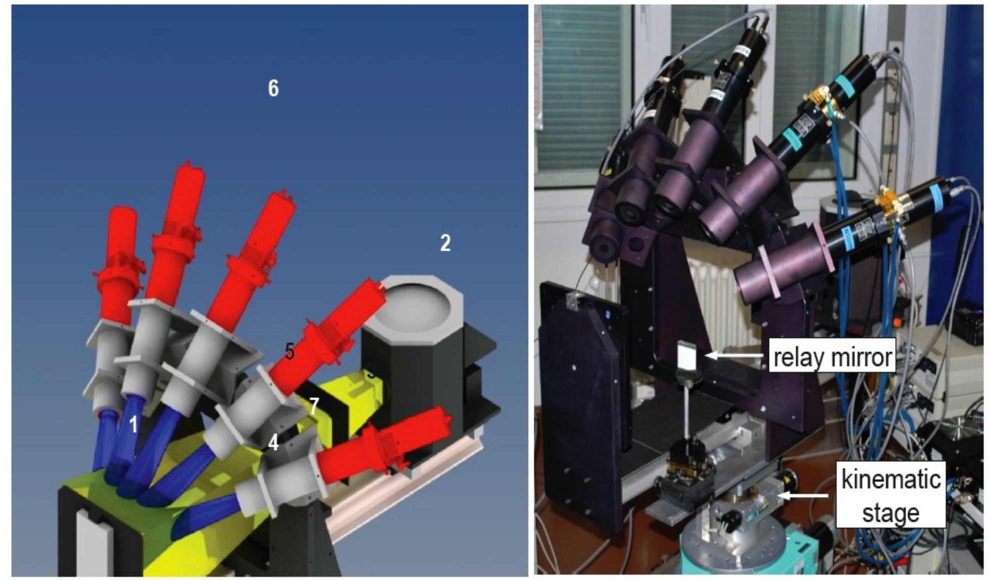
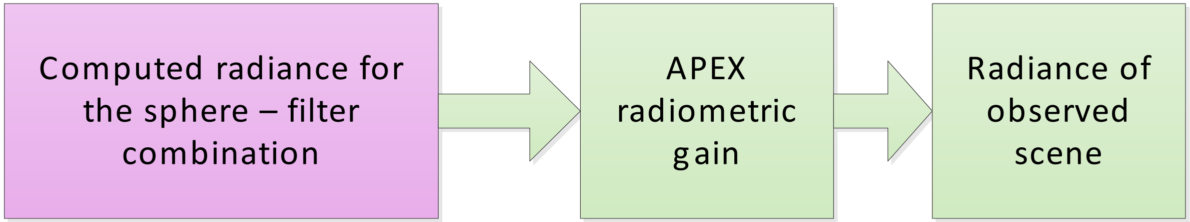


FIGURE 1. Mechanical set-up of RASTA (left): (1) Reflectance panel, (2) lamp housing, (3) translation stage, (4) mounting adapter, (5) radiometer holder, (6) radiometer, (7) sensor fitting. Yellow: beam illuminating reflectance panel, blue: field of view of radiometers. RASTA after its alignment for calibration in the 0°:45° viewing geometry at the SRCF of PTB (right).

Step 2: Writing down the calculation equations



$$L_{\text{sphere}} = DN_{\text{sphere}} G_{\text{SVC}}$$

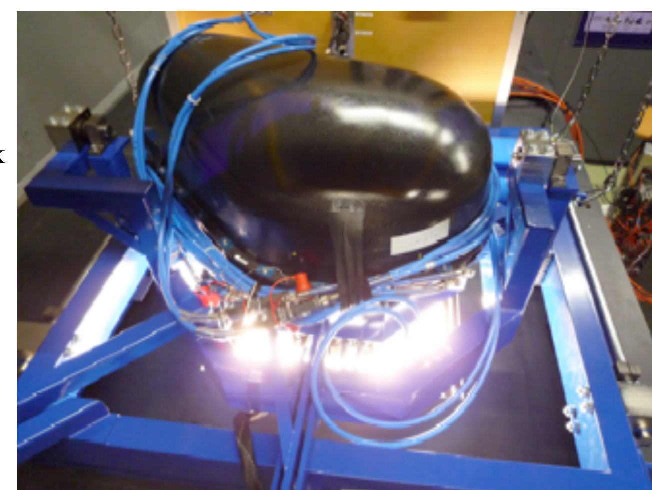
$$L_{\text{sph-filt}} = L_{\text{sphere}} \tau_{\text{filter}}$$

$$L_{\text{scene}} = G_{\text{APEX}} DN_{\text{APEX,scene}}$$

$$DN_{\text{APEX,scene}} = DN_{\text{APEX,scene,light}} - DN_{\text{APEX,scene,dark}}$$

$$G_{\text{APEX}} = L_{\text{sph-filt}} / DN_{\text{APEX,cal}}$$

$$DN_{\text{APEX,cal}} = DN_{\text{APEX,cal,light}} - DN_{\text{APEX,cal,dark}}$$

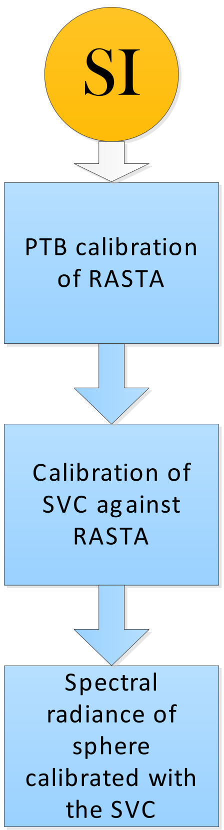


Step 3: Considering the sources of uncertainty

1. From the calculation equation

2. From assumptions in the comparison made

Step 3: Considering the sources of uncertainty



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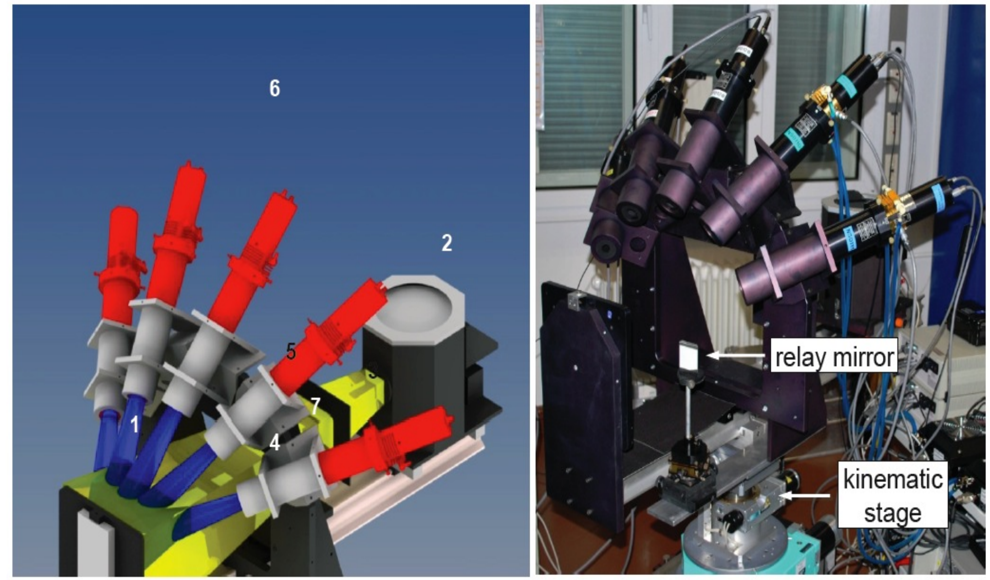

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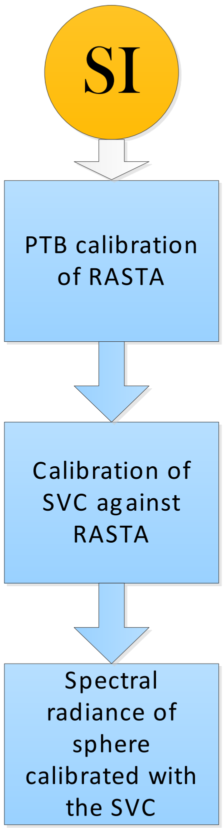
Step 3: Considering the sources of uncertainty



$$L_{\text{sphere}} = DN_{\text{sphere}} G_{\text{SVC}}$$

- Uncertainties associated with the terms in the calculation equation:
  - Signal: Noise in light signal, noise in dark signal
  - SVC gain: Calculated from the previous step (its calibration against RASTA)
- Uncertainties associated with the assumptions in the comparison:
  - Change in SVC between calibration and use
  - External stray light influencing the SVC differently
  - Internal stray light influencing the SVC differently
  - Environmental sensitivities of the SVC (temperature, pressure, humidity)
  - Linearity of the SVC

8 steps to an uncertainty budget

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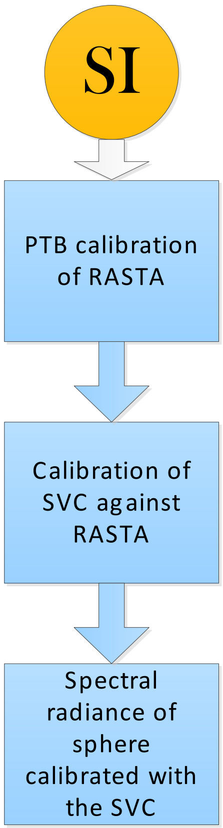
Step 4: Creating the measurement equation



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$$L_{\text{sphere}} = \left(DN_{\text{sphere\_l}} - DN_{\text{sphere\_d}}\right) G_{\text{SVC}} K_{\text{SVC\_dft}} K_{\text{stray}} K_{\text{stray\_in}} K_{\text{temp}} K_{\text{lin}}$$

Step 5: Determining the sensitivity coefficients

$$L_{\text{sphere}} = \left(DN_{\text{sphere\_l}} - DN_{\text{sphere\_d}}\right) G_{\text{SVC}} K_{\text{SVC\_dft}} K_{\text{stray}} K_{\text{stray\_in}} K_{\text{temp}} K_{\text{lin}}$$

$$\left(DN\right)_{\text{s}} = \frac{1}{N}\sum_{i=1}^{N}\left(DN\right)_{\text{light},i} - \frac{1}{M}\sum_{i=1}^{M}\left(DN\right)_{\text{dark},j}$$

$$c_{(DN)_{\text{light},i}} = \frac{\partial\left(DN\right)}{\partial\left(DN\right)_{\text{light},i}} = \frac{1}{N}, \quad i = 1,\ldots,N$$

$$c_{(DN)_{\text{dark},j}} = \frac{\partial\left(DN\right)}{\partial\left(DN\right)_{\text{dark},j}} = \frac{1}{M}, \quad j = 1,\ldots,M$$

$$u^2_{(DN)} = \sum_{i=1}^{N}\left(\frac{1}{N}\right)^2 u^2_{(DN)_{\text{light},i}} + \sum_{j=1}^{M}\left(\frac{1}{M}\right)^2 u^2_{(DN)_{\text{dark},j}}$$

$$u^2_{(DN)} = \left(\frac{u_{(DN)_{\text{light}}}}{\sqrt{N}}\right)^2 + \left(\frac{u_{(DN)_{\text{dark}}}}{\sqrt{M}}\right)^2$$

Step 5: Determining the sensitivity coefficients

$$L_{\text{sphere}} = \left(DN_{\text{sphere}}\right) G_{\text{SVC}} K_{\text{SVC\_dft}} K_{\text{stray}} K_{\text{stray\_in}} K_{\text{temp}} K_{\text{lin}}$$

$$c_{(DN)} = \frac{\partial L_{\text{sphere}}}{\partial (DN)} = \frac{L_{\text{sphere}}}{(DN)}$$

$$c_{G_{\text{SVC}}} = \frac{\partial L_{\text{sphere}}}{\partial G_{\text{SVC}}} = \frac{L_{\text{sphere}}}{G_{\text{SVC}}}$$

$$c_{K_i} = \frac{\partial L_{\text{sphere}}}{\partial K_i} = \frac{L_{\text{sphere}}}{K_i}$$

$$\left(\frac{u_{L_{\text{sph}}}}{L_{\text{sph}}}\right)^2 = \left(\frac{u_{(DN)}}{(DN)}\right)^2 + \left(\frac{u_{G_{\text{svc}}}}{G_{\text{svc}}}\right)^2 + \left(\frac{u_{K_{\text{svc-dft}}}}{K_{\text{svc-dft}}}\right)^2 + \text{K}$$

$$c_{K_i} = \frac{\partial L_{\text{sphere}}}{\partial K_i} = \frac{L_{\text{sphere}}}{K_i}$$

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Step 6: Assigning uncertainties

$$L_{\text{sphere}} = \left(DN_{\text{sphere}}\right) G_{\text{SVC}} K_{\text{SVC\_dft}} K_{\text{stray}} K_{\text{stray\_in}} K_{\text{temp}} K_{\text{lin}}$$

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The Law of Propagation of Uncertainties

$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

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Things you might already "know"

$$u_c^2(y) = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j)$$

Has a sensitivity coefficient
Adding in quadrature (% or units)

Averages reduce by $1/\sqrt{n}$

This term is to do with correlation

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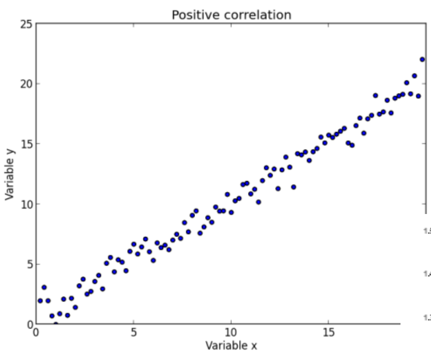
The Law of Propagation of Uncertainties

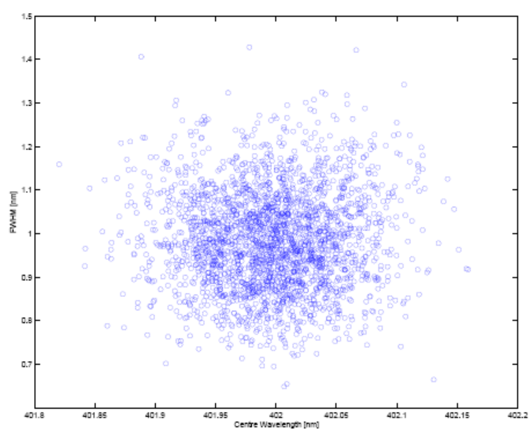
$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

Correlation: Type A and Type B methods

Correlation occurs when there is something in common between the different quantities, or between the measured values being combined.

Type A: From the data





Type B: From knowledge

$$E_i = E_{\mathrm{True}} + S + R_i$$

This is where the correlation comes from!

Systematic Effects!

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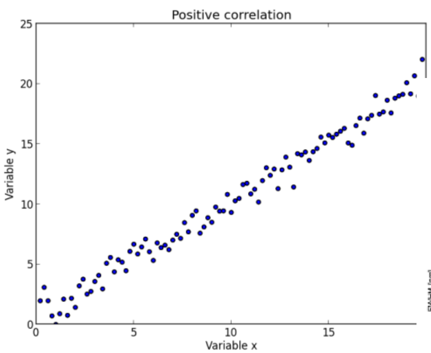
Correlation: Type A and Type B methods

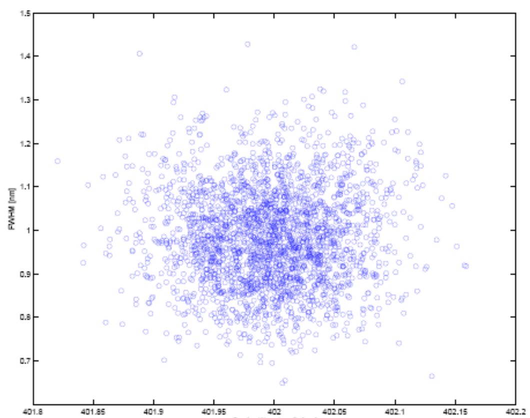
Correlation occurs when there is something in common between the different quantities, or between the measured values being combined.

Type A: From the data

$$r(x,y) = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{s_X}\right)\left(\frac{Y_i - \bar{Y}}{s_Y}\right)$$

$$u(x,y) = u(x)u(y)r(x,y)$$





$$u_c^2(y) = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j)$$

Correlation: Type A and Type B methods

Correlation occurs when there is something in common between the different parameters, or between the measurements being combined.

Calculate covariance

Remove covariance

Type B: From knowledge

$$E_i = E_{\text{True}} + S + R_i$$

This is where the correlation comes from!

Systematic

Effects!

Averaging partially correlated data

$$E_i = E_{\mathrm{True}} + S + R_i$$

$$E_{\mathrm{M}} = \frac{E_1 + E_2 + E_3}{3}$$

$$u(E_i) = u^2(S) + u^2(R_i)$$

$$\frac{\partial E_{\mathrm{M}}}{\partial E_1} = \frac{\partial E_{\mathrm{M}}}{\partial E_2} = \frac{\partial E_{\mathrm{M}}}{\partial E_3} = \frac{1}{3}$$

$$u(E_i, E_j) = u^2(S); \qquad i \neq j$$

$$u_{\mathrm{c}}^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

$$u^2(E_{\mathrm{M}}) = 3\left(\frac{1}{3}\right)^2 u^2(S) + 3\left(\frac{1}{3}\right)^2 u^2(R_i) + 3 \times 2 \times \left(\frac{1}{3}\right)^2 u^2(S)$$

$$u^2(E_{\mathrm{M}}) = (3+6)\left(\frac{1}{3}\right)^2 u^2(S) + 3\left(\frac{1}{3}\right)^2 u^2(R_i)$$

$$u^2(E_{\mathrm{M}}) = u^2(S) + \left(\frac{u(R_i)}{\sqrt{3}} \right)^2$$

Averaging partially correlated data

$$E_{\mathrm{M}} = \frac{E_1 + E_2 + E_3}{3}$$

$$E_i = E_{\mathrm{True}} + S + R_i$$

$$\frac{\partial E_{\mathrm{M}}}{\partial E_1} = \frac{\partial E_{\mathrm{M}}}{\partial E_2} = \frac{\partial E_{\mathrm{M}}}{\partial E_3} = \frac{1}{3}$$

$$E_{\mathrm{M}} = \frac{3E_{\mathrm{True}}}{3} + \frac{3S}{3} + \frac{R_1 + R_2 + R_3}{3}$$

$$u^2\left(E_{\mathrm{M}}\right) = u^2\left(S\right) + \left(\frac{u\left(R_i\right)}{\sqrt{3}}\right)^2$$

Systematic and random effects: Lamp measured 5 times (continued)

Systematic effects	Random effects
Reference calibration	Noise
Alignment	Lamp current fluctuation
Lamp current setting	
Temperature sensitivities	

$$S, u(S)$$

$$R_i, u(R_i)$$

$$u^2(E_\mathrm{M}) = u^2(S) + \left(\frac{u(R_i)}{\sqrt{n}}\right)^2$$

$$E_i = E_\mathrm{True} + S + R_i$$

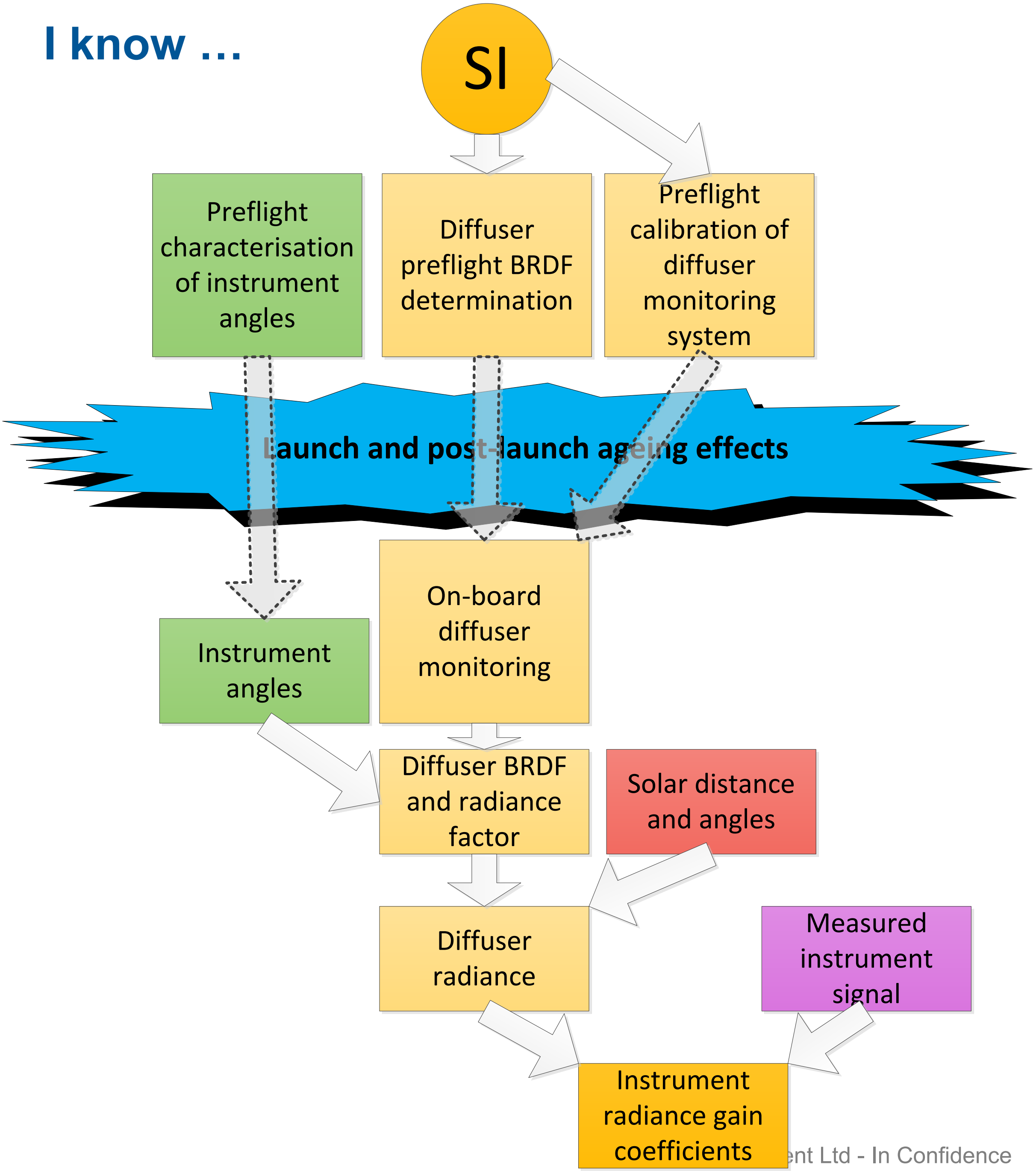
$$u^2(E_i) = u^2(S) + u^2(R_i)$$

Yes, Emma … But …

EO is different!

- You can’t write simple equations like that!
- We lose our traceability on launch!
- We can’t define our measurement model!

I know …



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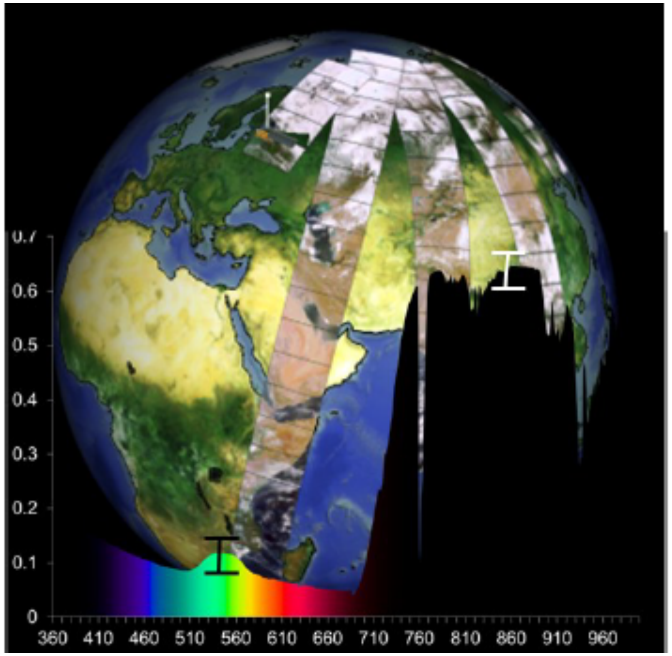
The first course (3rd/4th July)

Uncertainty Analysis
for **Earth Observation**
Radiometric Instrument Calibration



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- (Go to: http://www.npl.co.uk/events/3-4-jul-2014-uncertainties-for-earth-observation-training-course)

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