

# **Polarimetric Calibration of Spaceborne SAR Data in the Presence of the Ionosphere by Means of Azimuth Sub-bands**

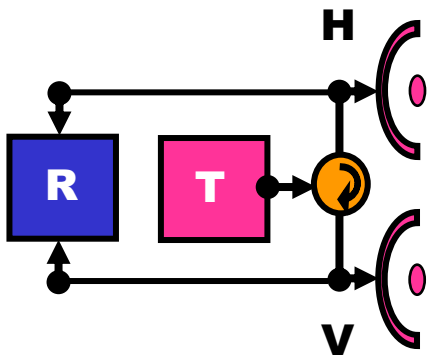
J-S. Kim and K. Papathanassiou

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**Microwaves and Radar Institute (DLR-HR)**

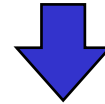


# Polarimetric System Distortion Model



**Tx / Rx Distortion:**

$$\begin{bmatrix} O_{HH} & O_{HV} \\ O_{VH} & O_{VV} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH} & R_{HV} \\ R_{VH} & R_{VV} \end{bmatrix}}_{\text{Receive}} \underbrace{\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}}_{\text{Transmit}} + \underbrace{\begin{bmatrix} T_{HH} & T_{HV} \\ T_{VH} & T_{VV} \end{bmatrix}}_{\text{Noise}} + \underbrace{\begin{bmatrix} N_{HH} & N_{HV} \\ N_{VH} & N_{VV} \end{bmatrix}}_{\text{Ambiguities}} + \underbrace{\begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix}}_{\text{Ambiguities}}$$



$$\begin{bmatrix} O_{HH} & O_{HV} \\ O_{VH} & O_{VV} \end{bmatrix} = Y \underbrace{\begin{bmatrix} 1 & \delta_{RH} \\ \delta_{RV} & f_R \end{bmatrix}}_{\text{Pol Dist. on Receive}} \underbrace{\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}}_{\text{Transmit}} \underbrace{\begin{bmatrix} 1 & \delta_{TH} \\ \delta_{TV} & f_T \end{bmatrix}}_{\text{Pol Dist. on Transmit}} + \begin{bmatrix} N_{HH} & N_{HV} \\ N_{VH} & N_{VV} \end{bmatrix} + \begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix}$$

**X-Talk Ratios: On Receive:**  $\delta_{RH} := \frac{R_{HV}}{R_{HH}}$      $\delta_{RV} := \frac{R_{VH}}{R_{HH}}$

On Transmit:  $\delta_{TH} := \frac{T_{HV}}{T_{HH}}$      $\delta_{TV} := \frac{T_{VH}}{T_{HH}}$

**H-V Channel Imbalance: On Receive:**  $f_R := \frac{R_{VV}}{R_{HH}}$

On Transmit:  $f_T := \frac{T_{VV}}{T_{HH}}$

**Abs. Gain on Tx & Rx:**  $Y := R_{HH} T_{HH}$

# Polarimetric System Distortion Model

System Tx / Rx Distortion + Propagation (i.e. Faraday Rotation) Distortion :

$$\begin{bmatrix} O_{HH} & O_{HV} \\ O_{VH} & O_{VV} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH} & R_{HV} \\ R_{VH} & R_{VV} \end{bmatrix}}_{\text{Receive}} \underbrace{\begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix}}_{\text{FR on Receive}} \underbrace{\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}}_{\text{FR on Transmit}} \underbrace{\begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix}}_{\text{FR on Transmit}} \underbrace{\begin{bmatrix} T_{HH} & T_{HV} \\ T_{VH} & T_{VV} \end{bmatrix}}_{\text{Transmit}} + \begin{bmatrix} N_{HH} & N_{HV} \\ N_{VH} & N_{VV} \end{bmatrix} + \begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix}$$

↓

$$\begin{bmatrix} O_{HH} & O_{HV} \\ O_{VH} & O_{VV} \end{bmatrix} = Y \underbrace{\begin{bmatrix} 1 & \delta_{RH} \\ \delta_{RV} & f_R \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix}}_{\text{Total distortion Matrix } [R^\Omega]} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix} \begin{bmatrix} 1 & \delta_{TH} \\ \delta_{TV} & f_T \end{bmatrix} + \begin{bmatrix} N_{HH} & N_{HV} \\ N_{VH} & N_{VV} \end{bmatrix} + \begin{bmatrix} A_{HH} & A_{HV} \\ A_{VH} & A_{VV} \end{bmatrix}$$

X-Talk – Faraday Rotation Ambiguity:

$$Y \begin{bmatrix} 1 & \delta_{XH} \\ \delta_{XV} & f_X \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix} = \begin{bmatrix} R_{HH}^\Omega & R_{HV}^\Omega \\ R_{VH}^\Omega & R_{VV}^\Omega \end{bmatrix}$$



Polar Decomposition of the X-talk matrix:

$$\begin{bmatrix} 1 & \delta_{XH} \\ \delta_{XV} & f_X \end{bmatrix} = \begin{bmatrix} 1 & \beta_{XH} \\ \beta_{XV} & f_X \end{bmatrix} \begin{bmatrix} \cos(\Omega_X) & \sin(\Omega_X) \\ -\sin(\Omega_X) & \cos(\Omega_X) \end{bmatrix}$$

$$\begin{bmatrix} R_{HH}^\Omega & R_{HV}^\Omega \\ R_{VH}^\Omega & R_{VV}^\Omega \end{bmatrix} = Y \begin{bmatrix} 1 & \beta_{XH} \\ \beta_{XV} & f_X \end{bmatrix} \begin{bmatrix} \cos(\Omega_X) & \sin(\Omega_X) \\ -\sin(\Omega_X) & \cos(\Omega_X) \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix} = Y \begin{bmatrix} 1 & \beta_{XH} \\ \beta_{XV} & f_X \end{bmatrix} \begin{bmatrix} \cos(\Omega + \Omega_X) & \sin(\Omega + \Omega_X) \\ -\sin(\Omega + \Omega_X) & \cos(\Omega + \Omega_X) \end{bmatrix}$$

# FR and X-Talk Rotation Ambiguity

## Total distortion Matrix:

$$\begin{bmatrix} R_{HH}^\Omega & R_{HV}^\Omega \\ R_{VH}^\Omega & R_{VV}^\Omega \end{bmatrix} = \begin{bmatrix} R_{HH} & R_{HV} \\ R_{VH} & R_{VV} \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix} = \begin{bmatrix} 1 & \beta_{XH} \\ \beta_{XV} & f_X \end{bmatrix} \begin{bmatrix} \cos(\Omega + \Omega_X) & \sin(\Omega + \Omega_X) \\ -\sin(\Omega + \Omega_X) & \cos(\Omega + \Omega_X) \end{bmatrix}$$

... there is an inherent ambiguity between  $\Omega$  (i.e. FR) and  $\Omega_X$  (i.e. rotation component of system dist.)

## Calibration error due to residual FR:

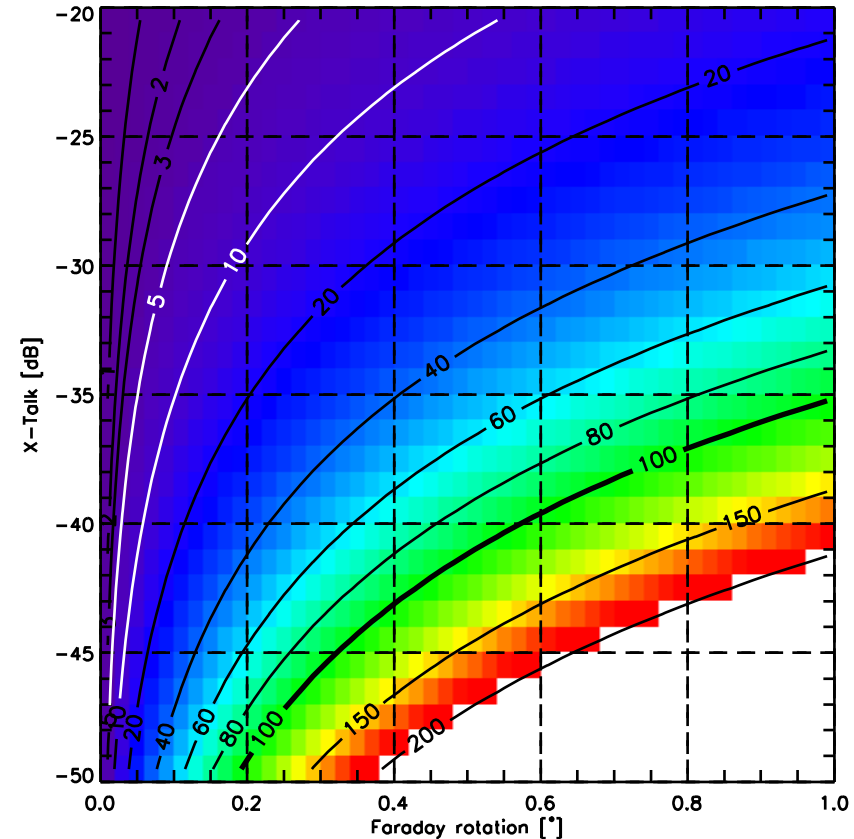
$$\begin{bmatrix} 1 & \delta_{EH} \\ \delta_{EV} & f_X \end{bmatrix} = k \begin{bmatrix} 1 & \delta_H \\ \delta_V & f_X \end{bmatrix} \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) \\ \sin(\Omega) & \cos(\Omega) \end{bmatrix}$$

$$= k \begin{bmatrix} \cos(\Omega) + \delta_H \sin(\Omega) & -\sin(\Omega) + \delta_H \cos(\Omega) \\ \delta_V \cos(\Omega) + f_X \sin(\Omega) & -\delta_V \sin(\Omega) + f_X \cos(\Omega) \end{bmatrix}$$

For small FR angles:  $\cos(\Omega) \approx 1$  and  $\sin(\Omega) \approx \Omega$

$$\delta_{EH} := -\sin(\Omega) + \delta_H \cos(\Omega) \approx -\Omega + \delta_H$$

$$\delta_{EV} := -f \sin(\Omega) + \delta_V \cos(\Omega) \approx f\Omega + \delta_V$$



# Solving the Polarimetric System Calibration Problem cont.

## Exploring the $\vec{B} \cdot \vec{k}(\theta_a)$ dependency

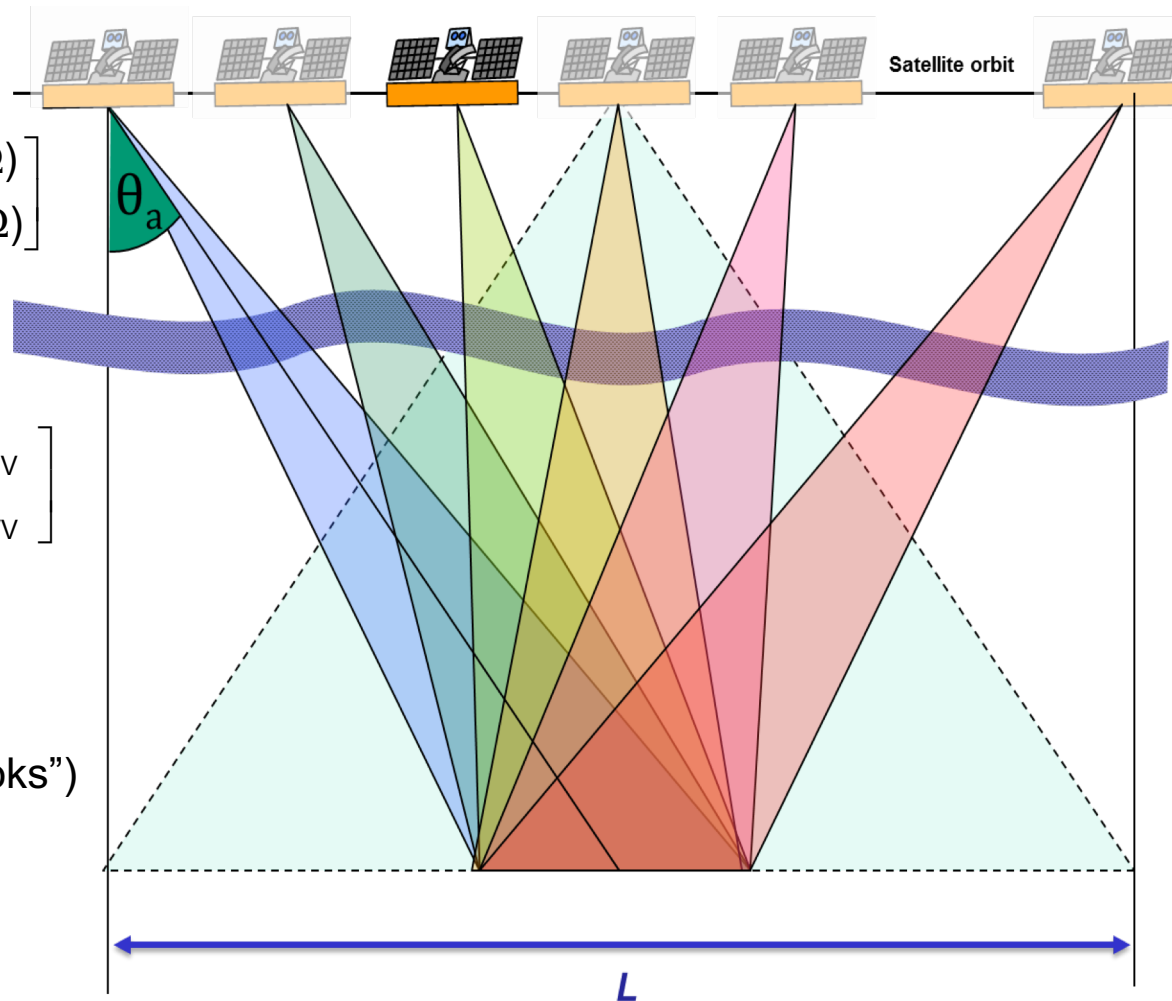
Distortion on receive:

$$\begin{bmatrix} R_{HH}^{\Omega} & R_{HV}^{\Omega} \\ R_{VH}^{\Omega} & R_{VV}^{\Omega} \end{bmatrix} = \begin{bmatrix} R_{HH} & R_{HV} \\ R_{VH} & R_{VV} \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix}$$

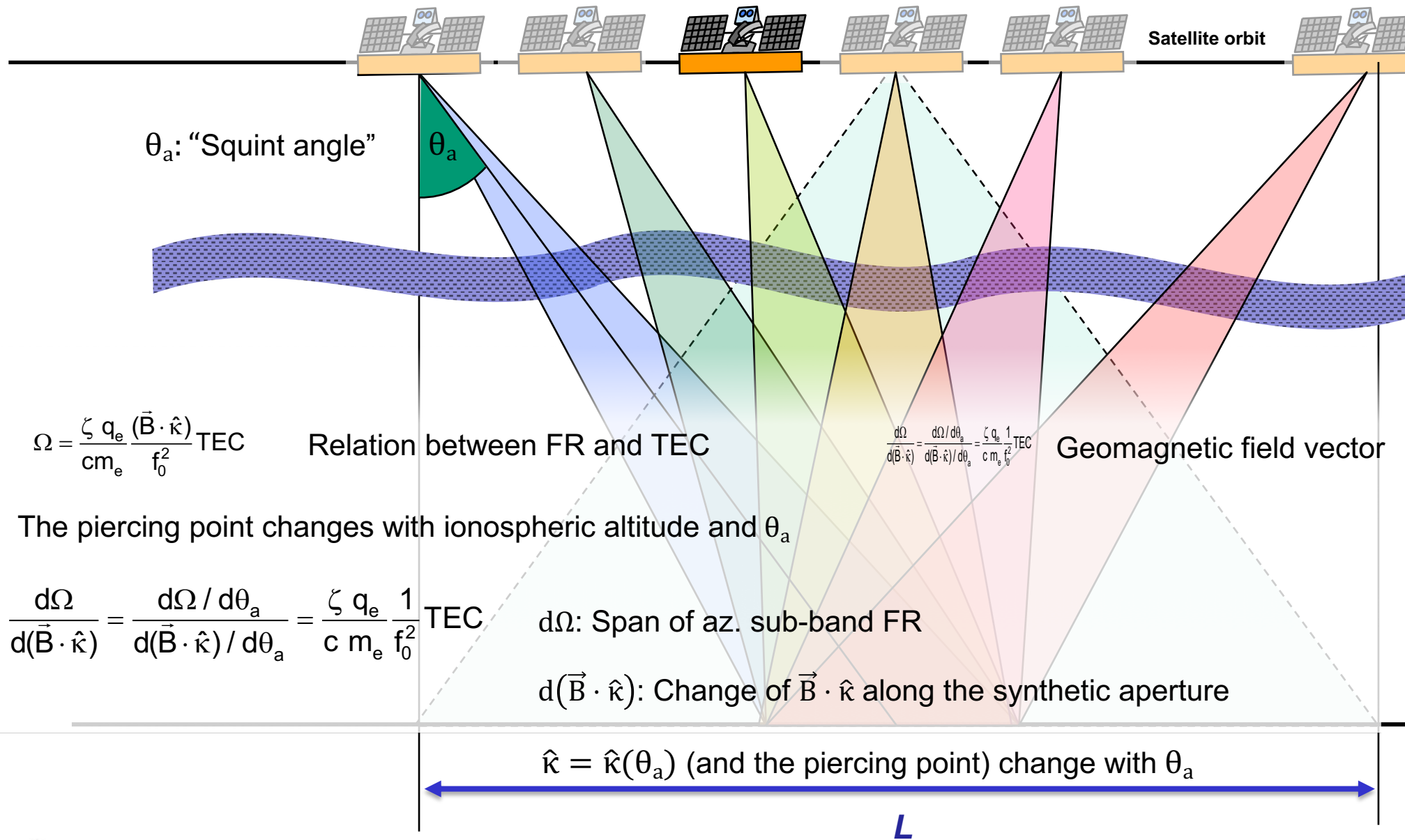
Distortion on transmit:

$$\begin{bmatrix} T_{HH}^{\Omega} & T_{HV}^{\Omega} \\ T_{VH}^{\Omega} & T_{VV}^{\Omega} \end{bmatrix} = \begin{bmatrix} \cos(\Omega) & \sin(\Omega) \\ -\sin(\Omega) & \cos(\Omega) \end{bmatrix} \begin{bmatrix} T_{HH} & T_{HV} \\ T_{VH} & T_{VV} \end{bmatrix}$$

... by exploring the variation across the synthetic aperture (i.e. across Azimuth "Looks")



# Azimuth Dependency of FR



# Separation of FR from X-Talk Rotation Component

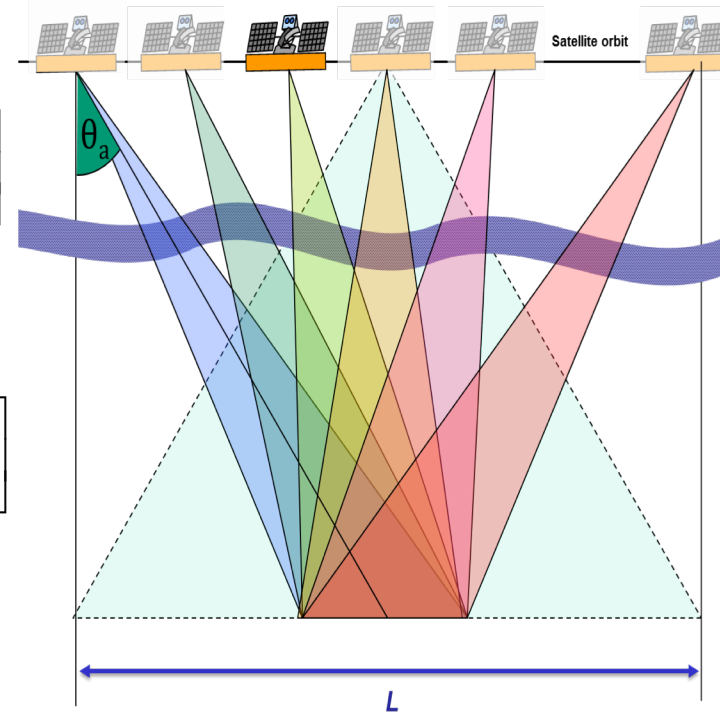
## 2) Exploring the difference in the azimuth dependency of the two components

Distortion on receive:

$$\begin{bmatrix} R_{HH}^{\Omega}(\theta_a) & R_{HV}^{\Omega}(\theta_a) \\ R_{VH}^{\Omega}(\theta_a) & R_{VV}^{\Omega}(\theta_a) \end{bmatrix} = \begin{bmatrix} R_{HH}(\theta_a) & R_{HV}(\theta_a) \\ R_{VH}(\theta_a) & R_{VV}(\theta_a) \end{bmatrix} \begin{bmatrix} \cos(\Omega(\theta_a)) & \sin(\Omega(\theta_a)) \\ -\sin(\Omega(\theta_a)) & \cos(\Omega(\theta_a)) \end{bmatrix}$$

Distortion on transmit:

$$\begin{bmatrix} T_{HH}^{\Omega}(\theta_a) & T_{HV}^{\Omega}(\theta_a) \\ T_{VH}^{\Omega}(\theta_a) & T_{VV}^{\Omega}(\theta_a) \end{bmatrix} = \begin{bmatrix} \cos(\Omega(\theta_a)) & \sin(\Omega(\theta_a)) \\ -\sin(\Omega(\theta_a)) & \cos(\Omega(\theta_a)) \end{bmatrix} \begin{bmatrix} T_{HH}(\theta_a) & T_{HV}(\theta_a) \\ T_{VH}(\theta_a) & T_{VV}(\theta_a) \end{bmatrix}$$



**FR variation across the synthetic aperture (L):**

$$\frac{d\Omega}{d(\vec{B} \cdot \hat{\kappa})} = \frac{d\Omega / d\theta_a}{d(\vec{B} \cdot \hat{\kappa}) / d\theta_a} = \frac{\zeta q_e}{c m_e} \frac{1}{f_0^2} \text{TEC}$$

$d\Omega$ : Span of az. sub-band FR

$d(\vec{B} \cdot \hat{\kappa})$ : Change of  $\vec{B} \cdot \hat{\kappa}$  along the synthetic aperture

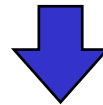
$\hat{\kappa} = \hat{\kappa}(\theta_a)$  (and the piercing point) change with  $\theta_a$

# Solving the Polarimetric System Calibration Problem cont.

Separation of FR from X-Talk Rotation by exploring the variation across the aperture (Az):

$$\begin{bmatrix} R_{HH}^{\Omega}(\theta_a) & R_{HV}^{\Omega}(\theta_a) \\ R_{VH}^{\Omega}(\theta_a) & R_{VV}^{\Omega}(\theta_a) \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH}(\theta_a) & R_{HV}(\theta_a) \\ R_{VH}(\theta_a) & R_{VV}(\theta_a) \end{bmatrix}}_{\text{Rx System Distortion}} \underbrace{\begin{bmatrix} \cos(\Omega(\theta_a)) & \sin(\Omega(\theta_a)) \\ -\sin(\Omega(\theta_a)) & \cos(\Omega(\theta_a)) \end{bmatrix}}_{\text{FR on Receive}}$$

$$\begin{bmatrix} R_{HH}(\theta_a) & R_{HV}(\theta_a) \\ R_{VH}(\theta_a) & R_{VV}(\theta_a) \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH}^I & R_{HV}^I \\ R_{VH}^I & R_{VV}^I \end{bmatrix}}_{\text{"Instrument" invariant in Az.}} \underbrace{\begin{bmatrix} R_{HH}^A(\theta_a) & R_{HV}^A(\theta_a) \\ R_{VH}^A(\theta_a) & R_{VV}^A(\theta_a) \end{bmatrix}}_{\text{"Antenna" variable in Az}}$$



Distortion on receive:

$$\begin{bmatrix} R_{HH}^{\Omega}(\theta_a) & R_{HV}^{\Omega}(\theta_a) \\ R_{VH}^{\Omega}(\theta_a) & R_{VV}^{\Omega}(\theta_a) \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH}^I & R_{HV}^I \\ R_{VH}^I & R_{VV}^I \end{bmatrix}}_{\text{"Instrument"}} \underbrace{\begin{bmatrix} R_{HH}^A(\theta_a) & R_{HV}^A(\theta_a) \\ R_{VH}^A(\theta_a) & R_{VV}^A(\theta_a) \end{bmatrix}}_{\text{Azimuth Variant}} \underbrace{\begin{bmatrix} \cos(\Omega(\theta_a)) & \sin(\Omega(\theta_a)) \\ -\sin(\Omega(\theta_a)) & \cos(\Omega(\theta_a)) \end{bmatrix}}_{\text{Azimuth Variant}}$$

Distortion on transmit:

$$\begin{bmatrix} T_{HH}^{\Omega}(\theta_a) & T_{HV}^{\Omega}(\theta_a) \\ T_{VH}^{\Omega}(\theta_a) & T_{VV}^{\Omega}(\theta_a) \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\Omega(\theta_a)) & \sin(\Omega(\theta_a)) \\ -\sin(\Omega(\theta_a)) & \cos(\Omega(\theta_a)) \end{bmatrix}}_{\text{Azimuth Variant}} \underbrace{\begin{bmatrix} T_{HH}^A(\theta_a) & T_{HV}^A(\theta_a) \\ T_{VH}^A(\theta_a) & T_{VV}^A(\theta_a) \end{bmatrix}}_{\text{Azimuth Variant}} \underbrace{\begin{bmatrix} T_{HH}^I & T_{HV}^I \\ T_{VH}^I & T_{VV}^I \end{bmatrix}}_{\text{"Instrument"}}$$



# Separation of FR from X-Talk Rotation component cont.

Assumption: Antenna patterns characterise the Az. variable system term:

$$\begin{bmatrix} R_{HH}(\theta_a) & R_{HV}(\theta_a) \\ R_{VH}(\theta_a) & R_{VV}(\theta_a) \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH}^I & R_{HV}^I \\ R_{VH}^I & R_{VV}^I \end{bmatrix}}_{\text{"Instrument" invariant in Az.}} \underbrace{\begin{bmatrix} R_{HH}^A(\theta_a) & R_{HV}^A(\theta_a) \\ R_{VH}^A(\theta_a) & R_{VV}^A(\theta_a) \end{bmatrix}}_{\text{"Antenna" variable in Az}}$$

FR variation across the synthetic aperture (L):

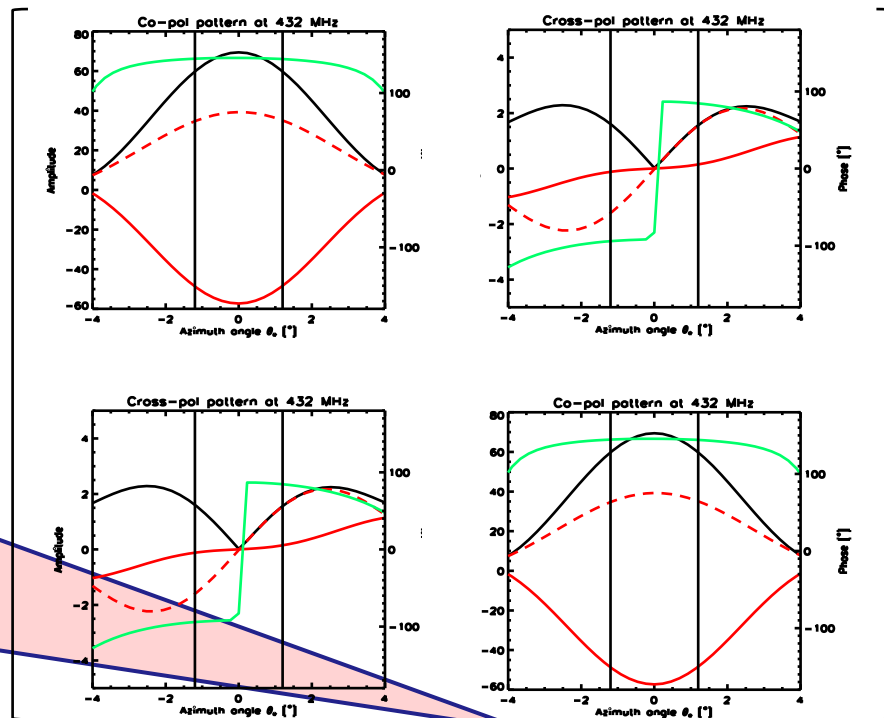
$$\frac{d\Omega}{d(\vec{B} \cdot \hat{r})} = \frac{d\Omega / d\theta_a}{d(\vec{B} \cdot \hat{r}) / d\theta_a} = \frac{\zeta q_e}{c m_e f_0^2} \text{TEC}$$

$d\Omega$ : Span of az. sub-band FR

$d(\vec{B} \cdot \hat{r})$ : Change of  $\vec{B} \cdot \hat{r}$  along the synthetic aperture

$\hat{r} = \hat{r}(\theta_a)$  (and the piercing point) change with  $\theta_a$

$$\begin{bmatrix} R_{HH}^A(\theta_a) & R_{HV}^A(\theta_a) \\ R_{VH}^A(\theta_a) & R_{VV}^A(\theta_a) \end{bmatrix} =$$



Distortion on receive:

$$\begin{bmatrix} R_{HH}^\Omega(\theta_a) & R_{HV}^\Omega(\theta_a) \\ R_{VH}^\Omega(\theta_a) & R_{VV}^\Omega(\theta_a) \end{bmatrix} = \underbrace{\begin{bmatrix} R_{HH}^I & R_{HV}^I \\ R_{VH}^I & R_{VV}^I \end{bmatrix}}_{\text{Instrument}} \underbrace{\begin{bmatrix} R_{HH}^A(\theta_a) & R_{HV}^A(\theta_a) \\ R_{VH}^A(\theta_a) & R_{VV}^A(\theta_a) \end{bmatrix}}_{\text{Antenna}} \underbrace{\begin{bmatrix} \cos(\Omega(\theta_a)) & \sin(\Omega(\theta_a)) \\ -\sin(\Omega(\theta_a)) & \cos(\Omega(\theta_a)) \end{bmatrix}}_{\text{Azimuth Variant}}$$

# Test Site: Remningstorp, Sweden (ALOS-1)

	ALPSRP	DATE
Descending	01697 2430	06/05/20
	02368 2430	06/07/05
	03039 2430	06/08/20
	03710 2430	06/10/05
	ALPSRP	DATE
Ascending	01908 1170	06/06/03
	02579 1170	06/07/19
	03921 1170	06/10/19
	04592 1170	06/12/04

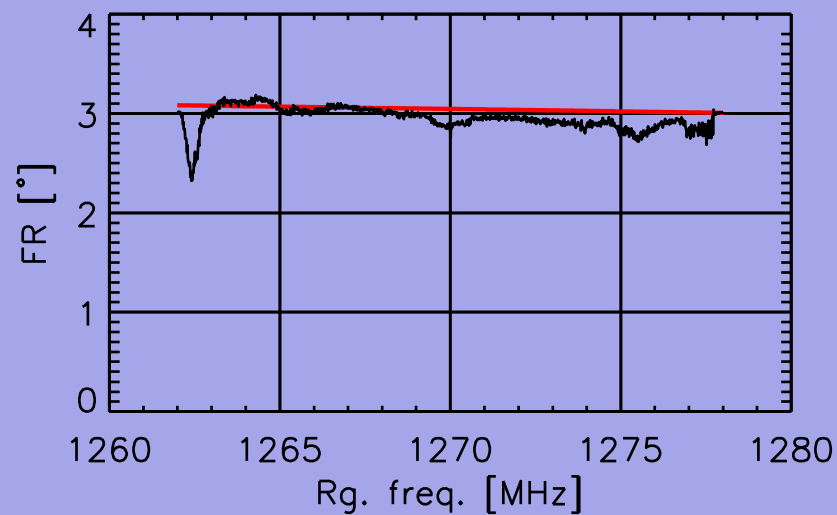
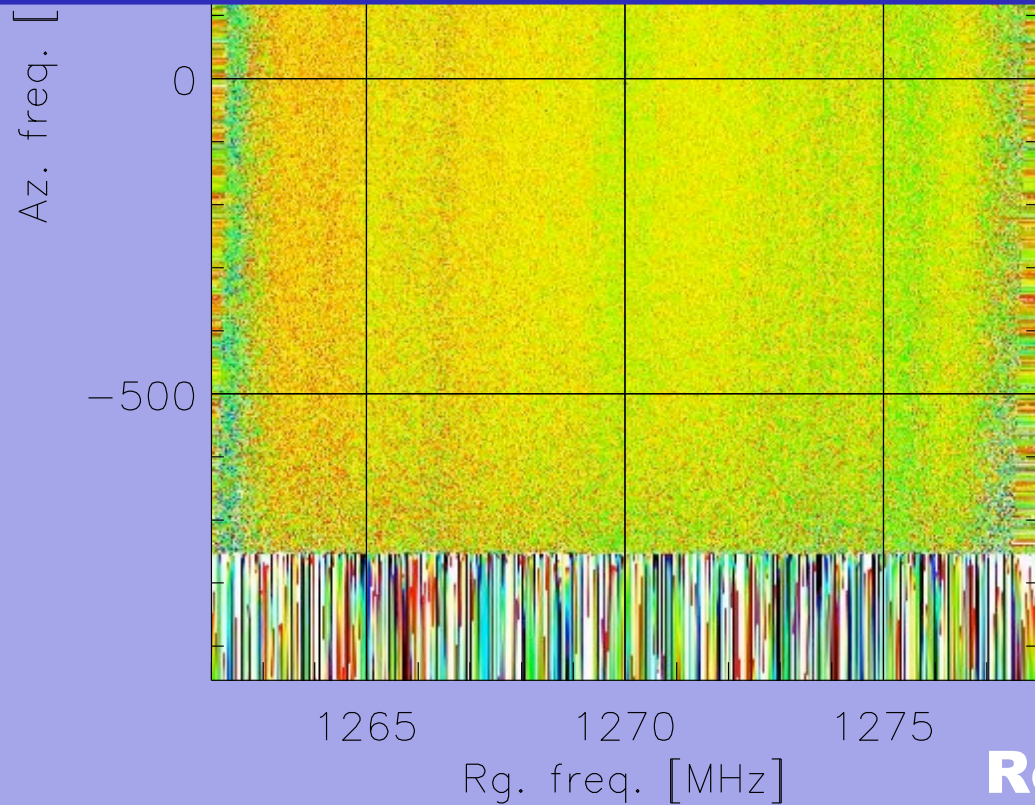
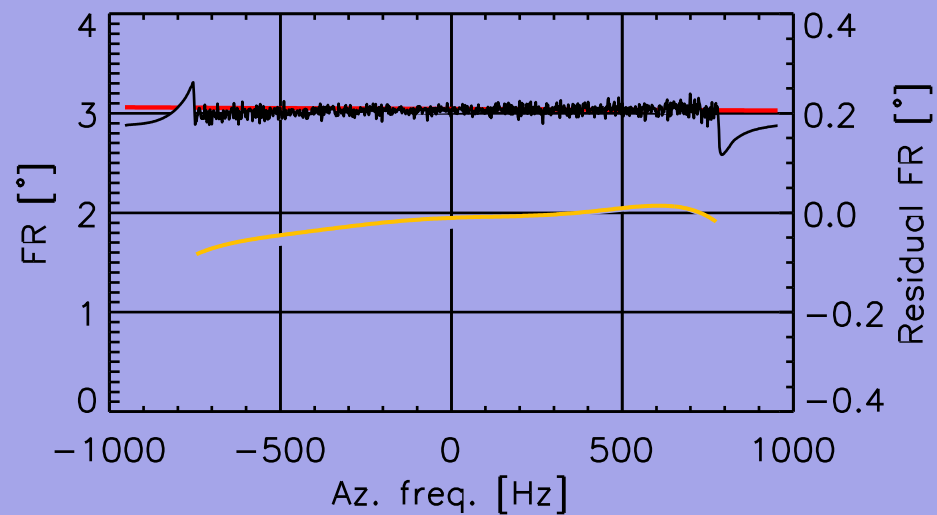
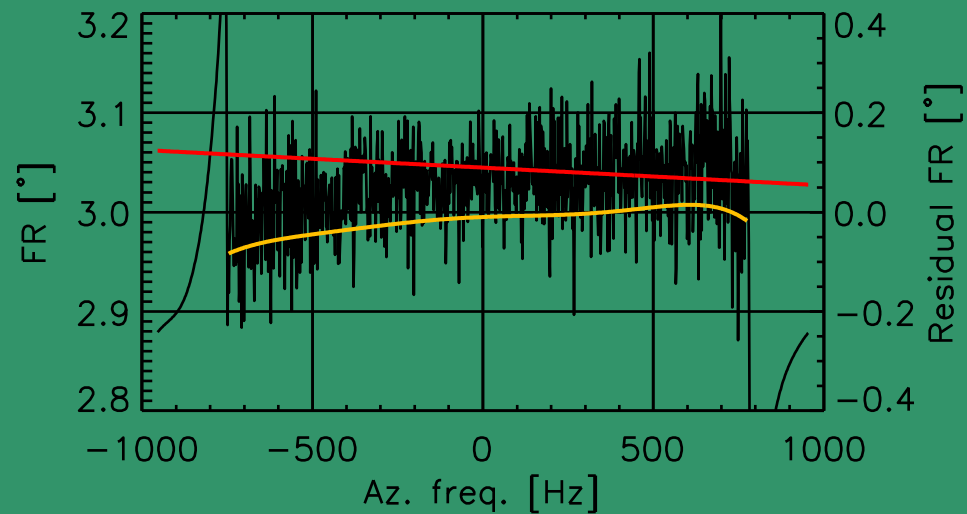
Azimuth ↑  
Range →

Range ↙  
Azimuth ↘

Range ←  
Azimuth ↓

ALPSRP	DATE
02295 2640	06/06/30
02966 2640	06/08/15
03637 2640	06/09/30
04308 2640	06/11/15
04979 2640	06/12/31
05650 2640	07/02/15
06321 2640	07/04/02
10347 2640	08/01/03
11689 2640	08/04/04
13702 2640	08/08/20

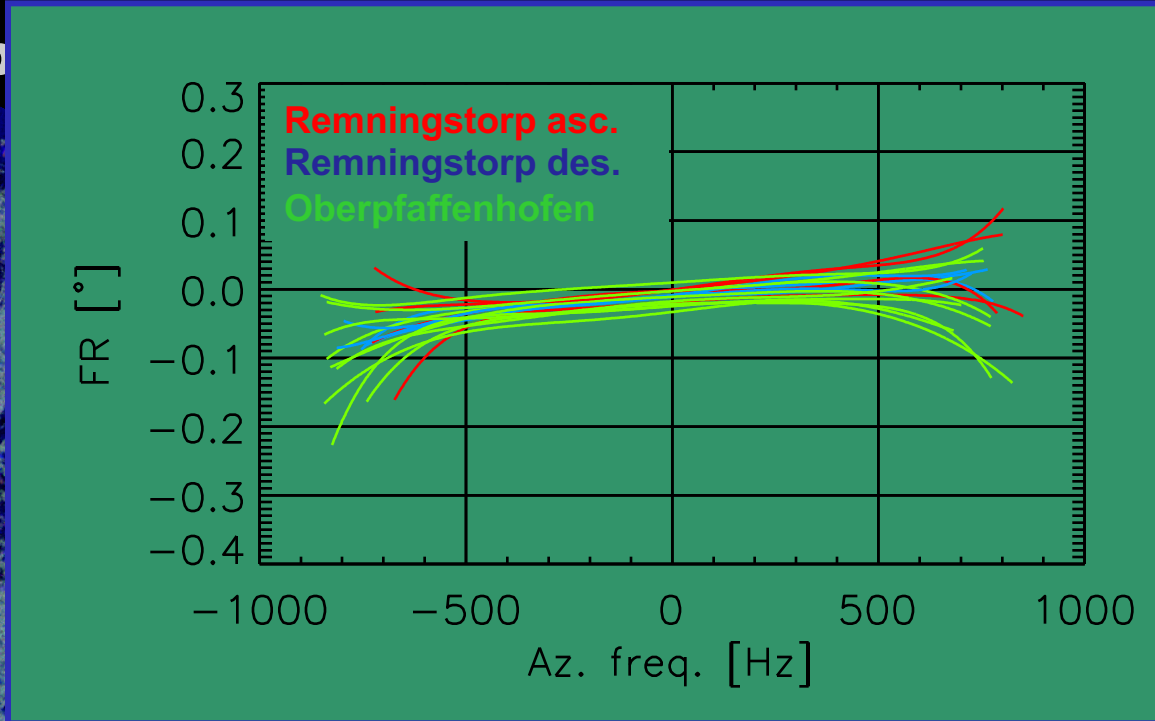
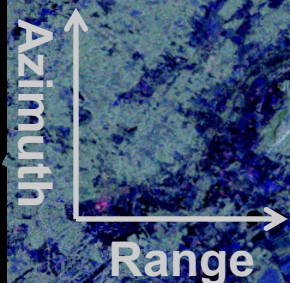
# Test Site: Oberpfaffenhofen, Germany, ALOS-1



**Remningstorp, Sweden (ALOS-1)**

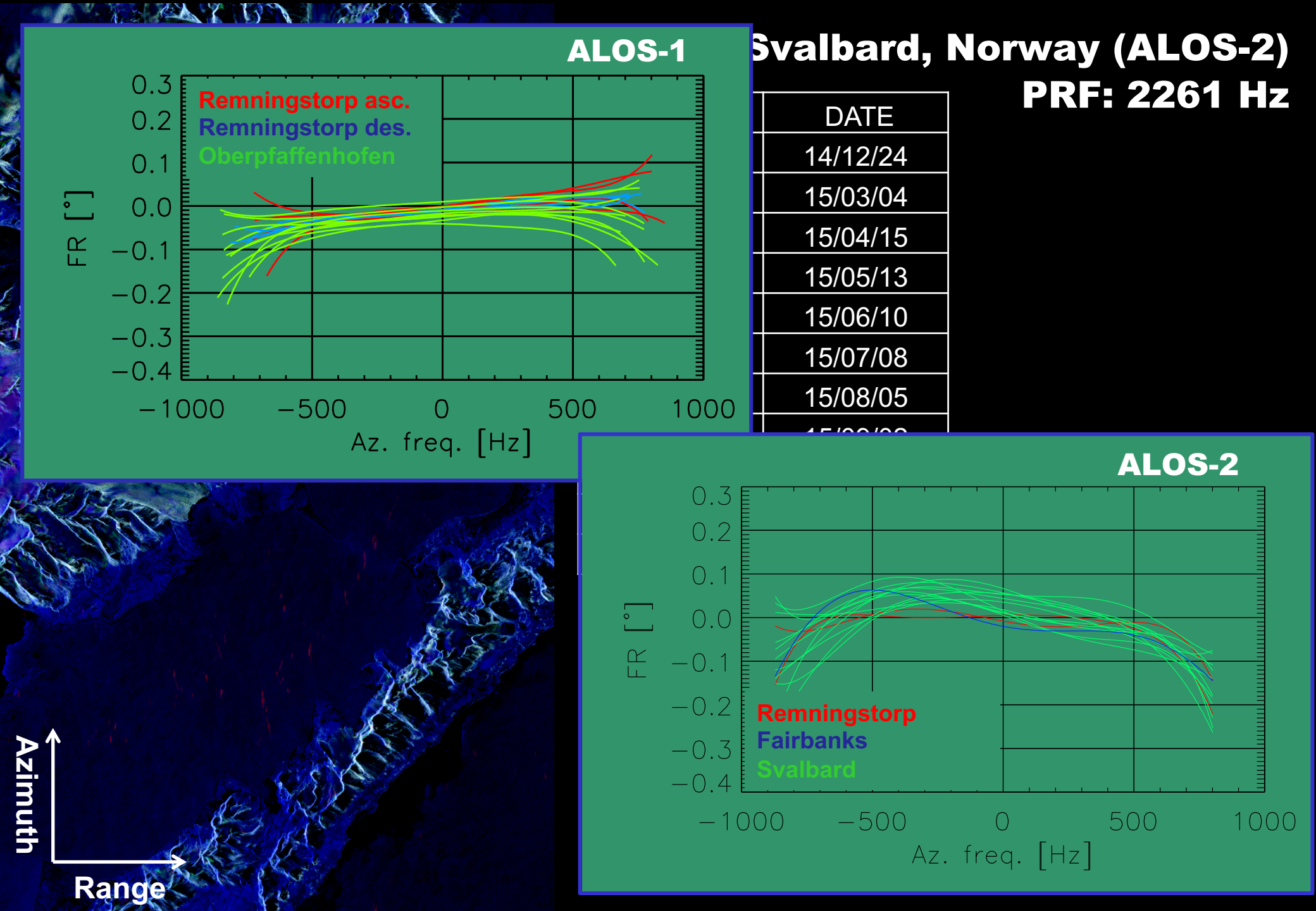
# Test Site: Remningstorp

	ALPSRP	DATE
Descending	01697 2430	06/05/20
	02368 2430	06/07/05
	03039 2430	06/08/20
	03710 2430	06/10/05
	ALPSRP	
Ascending	01908 1170	06/06/03
	02579 1170	06/07/19
	03921 1170	06/10/19
	04592 1170	06/12/04



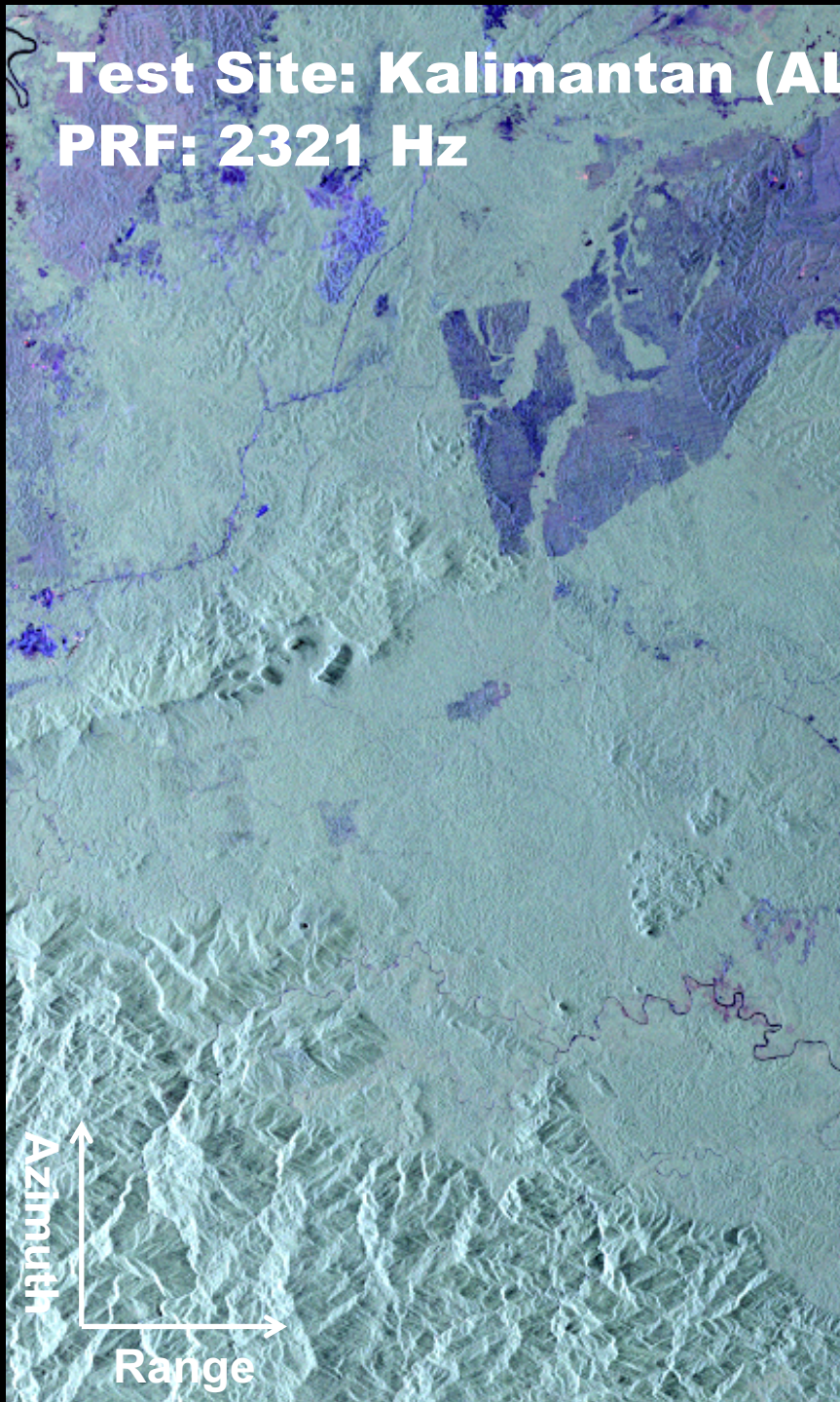
02966 2640	06/08/15
03637 2640	06/09/30
04308 2640	06/11/15
04979 2640	06/12/31
05650 2640	07/02/15
06321 2640	07/04/02
10347 2640	08/01/03
11689 2640	08/04/04
13702 2640	08/08/20

# Test Site: Oberpfaffenhofen, Germany, ALOS-1



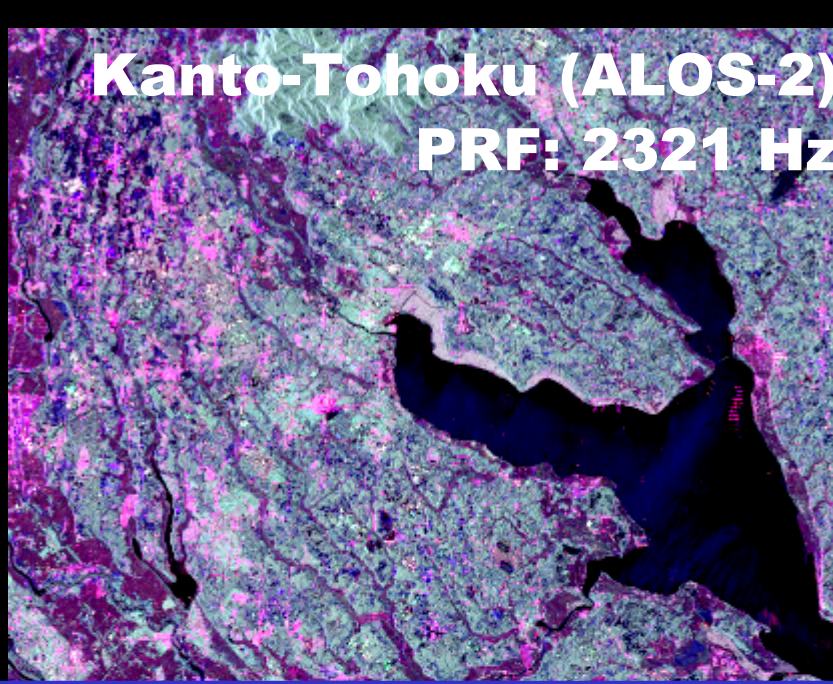
# Test Site: Kalimantan (ALOS-2)

PRF: 2321 Hz

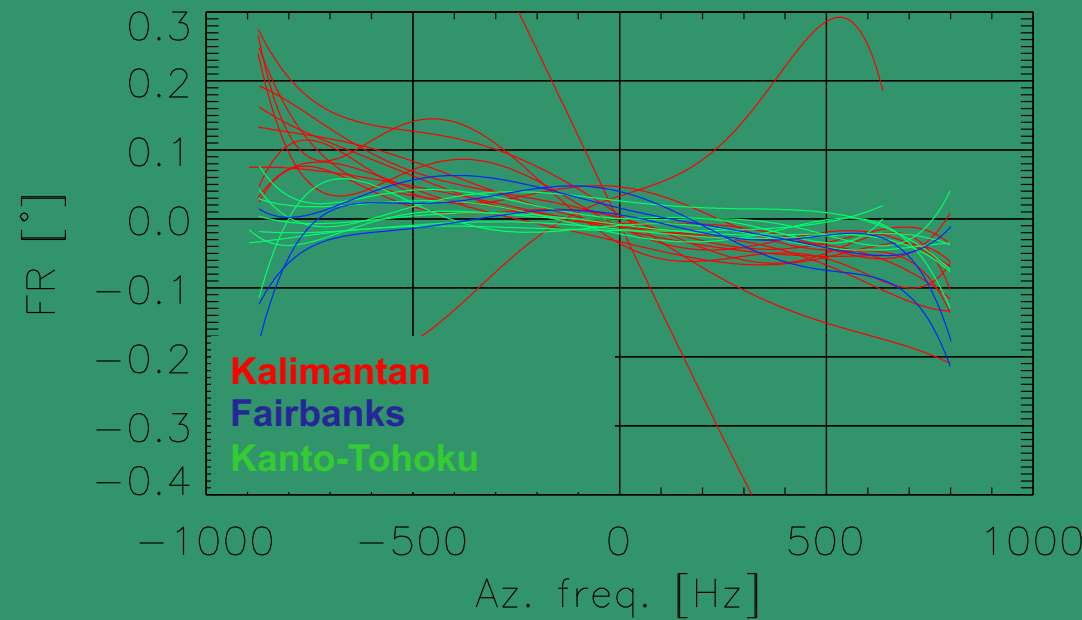


# Kanto-Tohoku (ALOS-2)

PRF: 2321 Hz

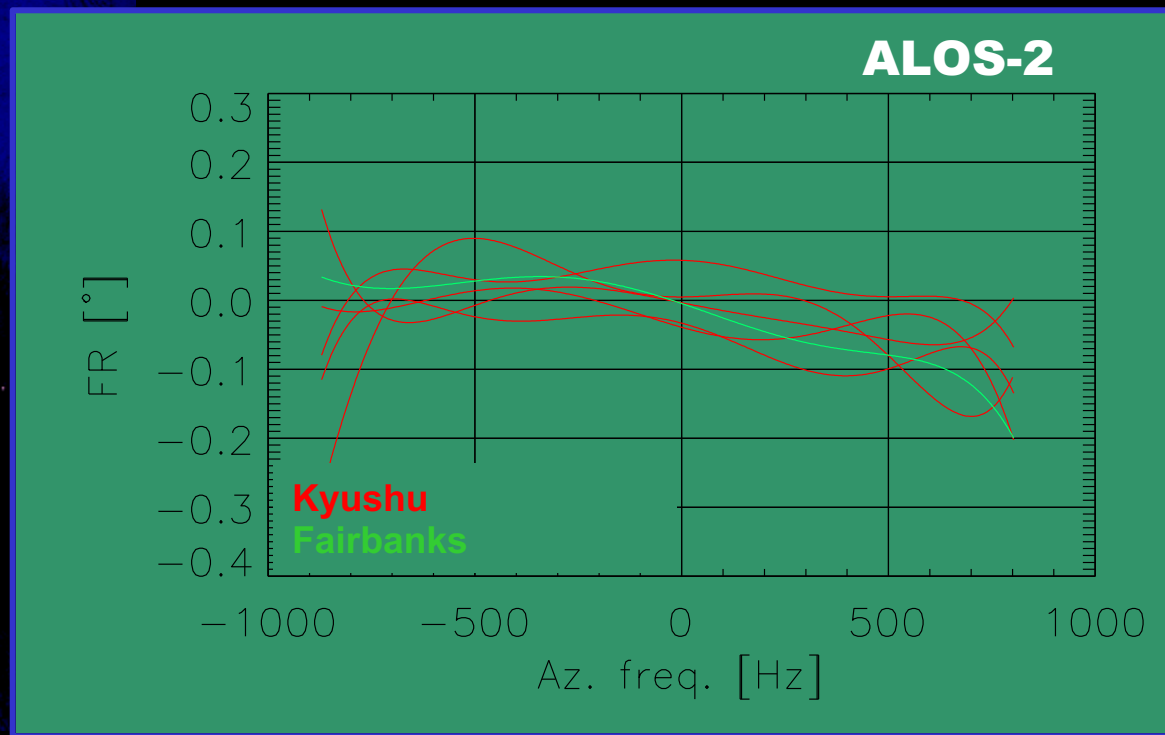
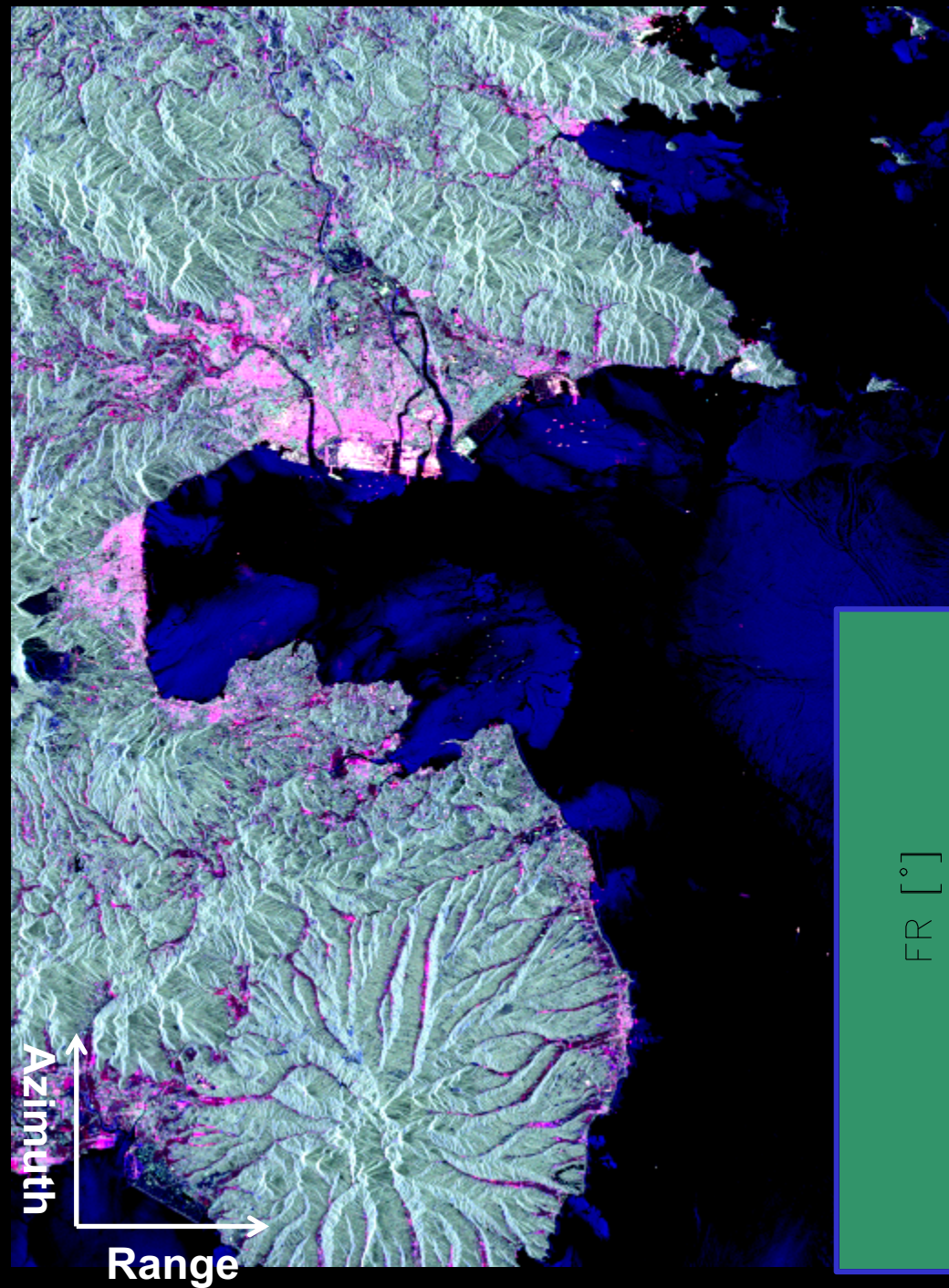


## ALOS-2



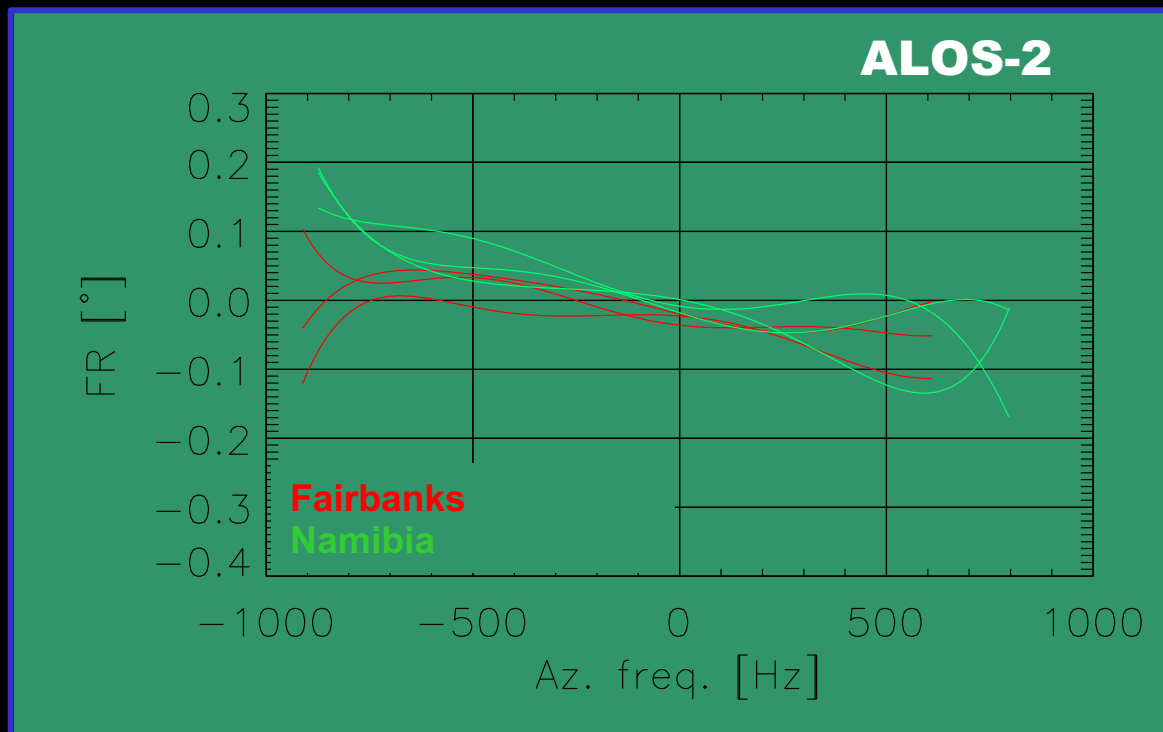
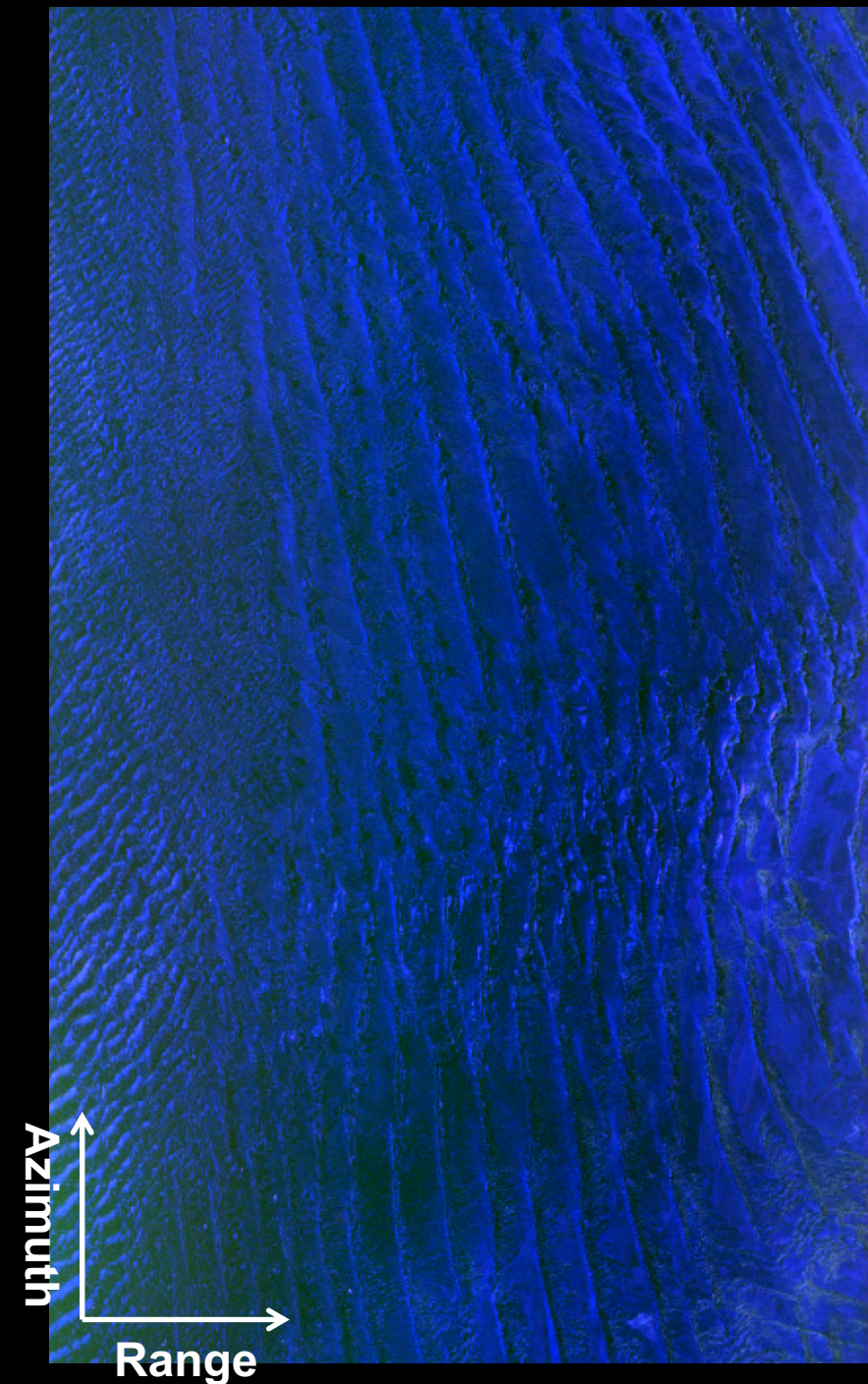
# Test Site: Kyushu (ALOS-2)

## PRF: 2610 Hz



**Test Site: Namibia (ALOS-2)**

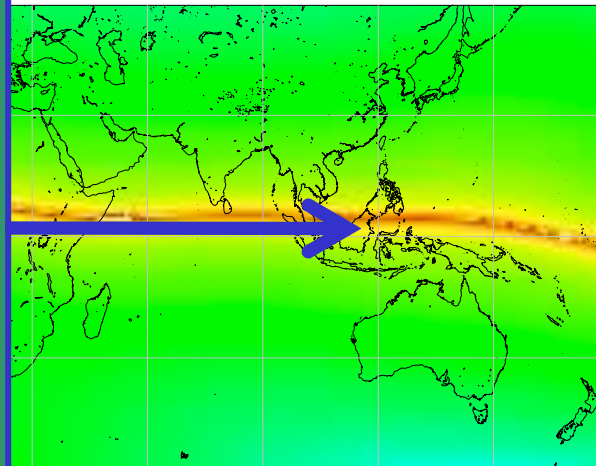
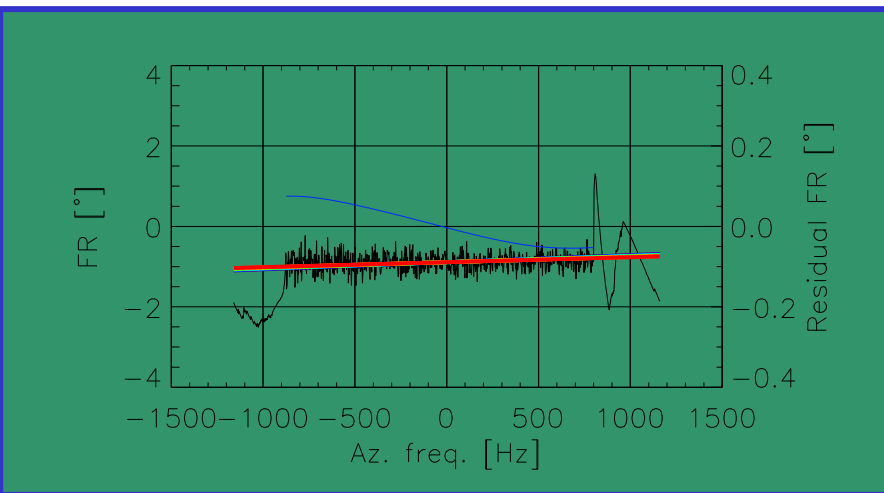
**PRF: 2596 Hz**





# Dependencies on Model Height

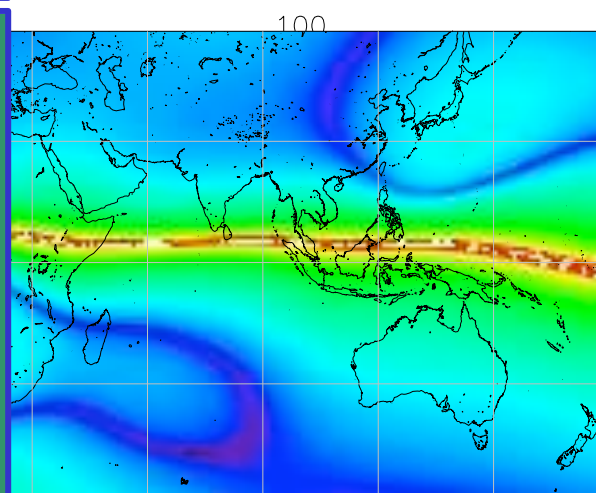
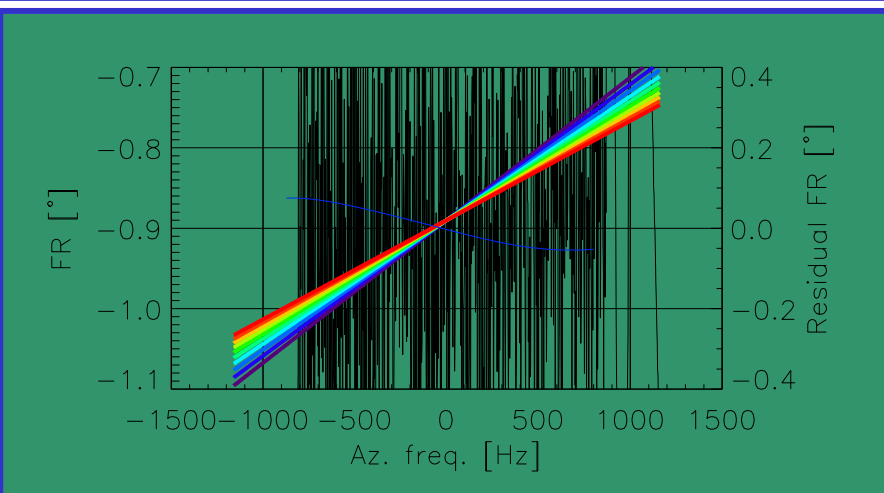
➤ The squint dependency of FR is a function of (geomagnetic) latitude, and altitude



*Azimuth dependent FR at extrema of bandwidth (@ 300 km alt.)*

$$\frac{d\Omega}{d\beta} \cdot \frac{\beta_0}{2}$$

*( $\beta_0$ : beam width)*

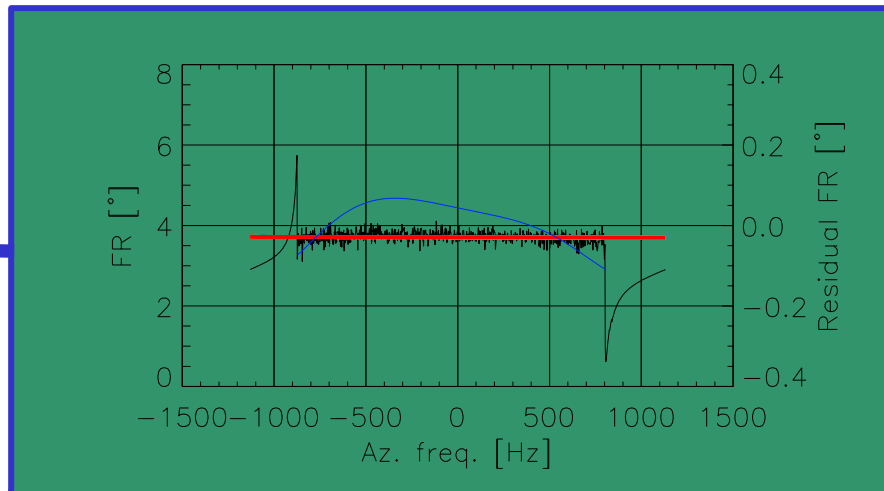
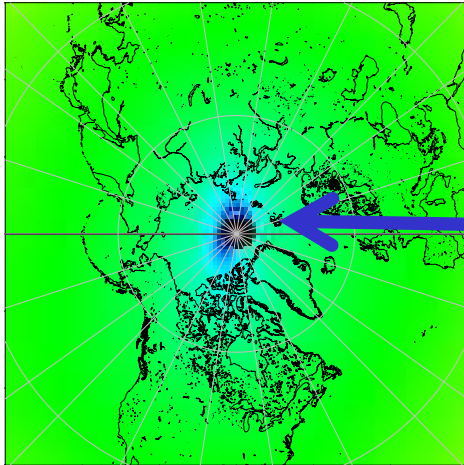


*Span of residual FR at extrema of bandwidth for altitude change*

$$\max\left(\frac{d\Omega}{d\beta} \cdot \frac{\beta_0}{2}\right) - \min\left(\frac{d\Omega}{d\beta} \cdot \frac{\beta_0}{2}\right)$$

# Dependencies on Height

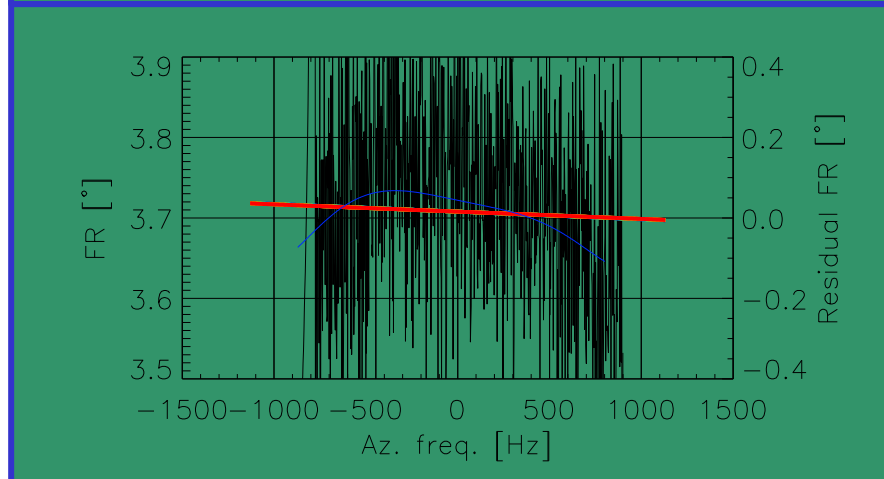
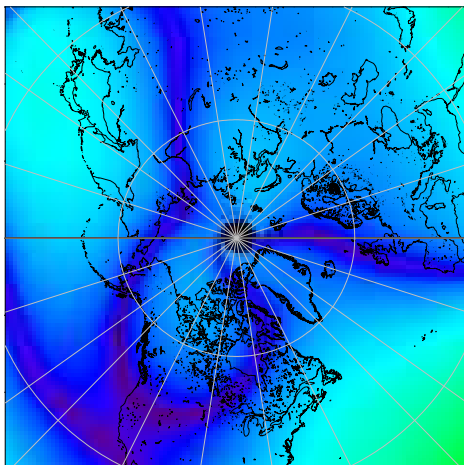
➤ The squint dependency of FR is a function of (geomagnetic) latitude, and altitude



*Azimuth dependent FR at  
extrema of bandwidth*

$$\frac{d\Omega}{d\beta} \cdot \frac{\beta_0}{2}$$

*( $\beta_0$ : beam width)*

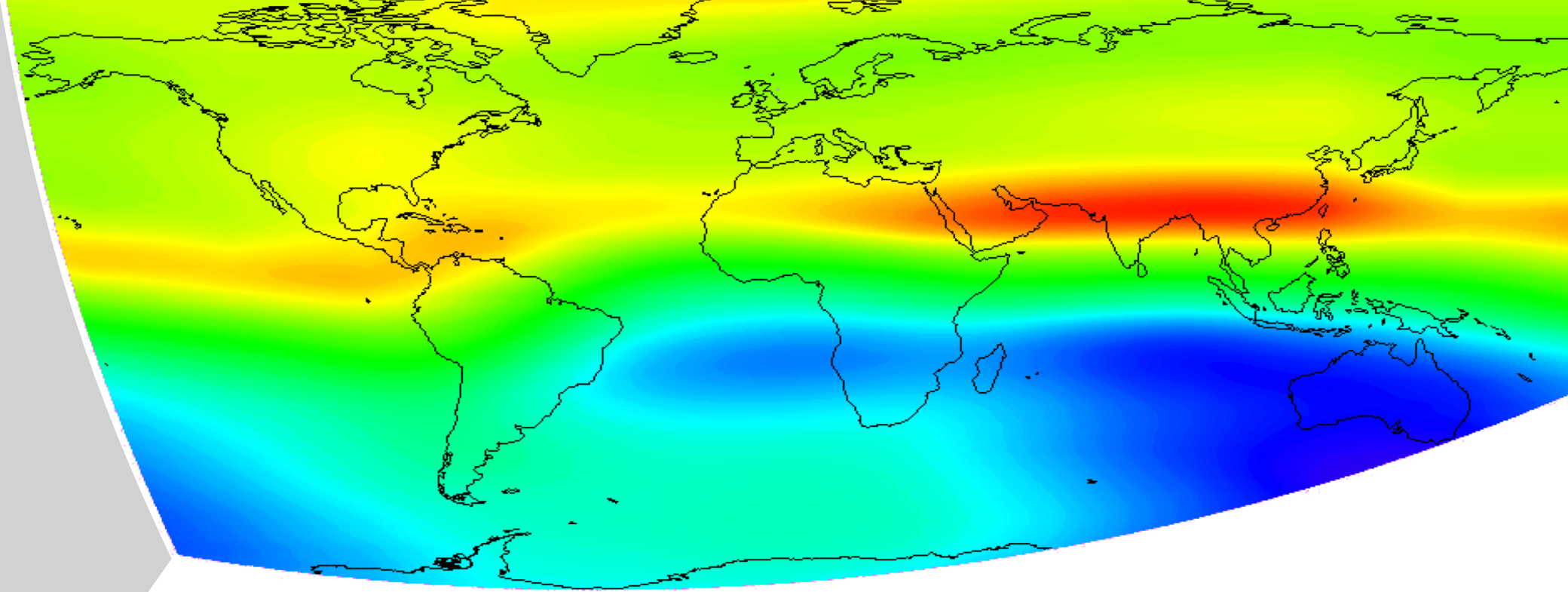


*Span of residual FR at  
extrema of bandwidth for  
altitude change*

$$\max\left(\frac{d\Omega}{d\beta} \cdot \frac{\beta_0}{2}\right) - \min\left(\frac{d\Omega}{d\beta} \cdot \frac{\beta_0}{2}\right)$$

# Conclusions

- There is an **inherent ambiguity** between FR distortion and the rotation component of the system distortion (x-talk);
- The azimuth variation of the FR provides a way to separate the FR distortion from system induced distortion for the calibration of spaceborne polarimetric data
- Determination of the azimuth dependency of FR relies on mean FR estimates and  $\vec{B} \cdot \hat{r}$ .
- At low latitudes  $\vec{B} \cdot \hat{r}$  becomes too sensitive with respect to the ionospheric height assumption.
- The proposed algorithm relaxes significantly the constraints imposed by conventional polarimetric calibration approaches in the sense that:
  - 1) there are no calibration devices (i.e. reflectors, transponders, etc.) required at the geomagnetic equator but they can still be located in areas where  $\mathbf{B} \cdot \mathbf{k} = 0$ ;
  - 2) it allows the estimation of X-Talk levels from each single scene and not only where  $\mathbf{B} \cdot \mathbf{k} = 0$  is fulfilled.
- Performance analysis (e.g. investigating scene-dependent deviation, etc.) to be done.



# **Polarimetric Calibration of Spaceborne SAR Data in the Presence of the Ionosphere by Means of Azimuth Sub-bands**

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